

Forecasting Russian renewable, nuclear, and total energy consumption using improved nonlinear grey Bernoulli model

Hsiao-Tien Pao¹, Hsin-Chia Fu², Hsiao-Cheng Yu³

¹ Department of Management Science, National Chiao Tung University,
Taiwan, ROC

² College of Engineering, Huaqiao University,
Quanzhou, Fujian 362021, China

³ Graduate Institute of Technology Management, National Chiao Tung University,
Taiwan, ROC

Abstract

Forecasts of renewable, nuclear, and total primary energy consumption are essential for a green energy system and the understanding of climate change in a rapidly growing market such as Russia. In this paper, nonlinear grey Bernoulli with power j model (NGBM ^{j}) is applied to predict these three different types of energy consumption. A numerical iterative method to optimize the powers of NGBM using mathematical software is also proposed. The NGBM with optimal power model is named NGBM^{op}. The forecasting ability of NGBM^{op} has remarkably improved, comparing with the grey model. For each time series, the best NGBM^{op} provides an accurate and reliable multi-step prediction with a MAPE value of less than 2.90 during the out-of-sample period of 2004-2009. The prediction results show that Russia's compound annual renewable, nuclear, and total energy consumption growth rates are set respectively at 1.95%, 2.44%, and 0.88% between 2010 and 2015.

Keywords: Grey prediction model; Nonlinear grey Bernoulli model; Nuclear; Renewable; Russia.

1. Introduction

A good forecasting technique is prerequisite for studies of green energy systems, not only for the cost-effectiveness of investment planning but also for the monitoring of environmental issues as well as demand side management. Forecasting studies for different types of energy consumption, e.g. renewable, nuclear, and total primary energy consumption, constitute a pivotal part of green energy policies, especially for an emerging market like Russia. Russia spreads out over a vast swath of land from the searing pre-Caspian deserts to the Arctic tundra and extends across 11 time zones (GMT+2 to GMT+12). The

impact of climate change, including the adverse socio-economic consequences of natural hazards, plays a critical role in the spatial and economic development of the country [1]. Therefore, an accurate prediction model is necessary for the clean energy system in Russia.

The prediction method includes multivariate models, univariate time-series models, and nonlinear intelligent models. A limitation of multivariate models is that their predictive ability depends on the availability and reliability of the independent variables data. Univariate time-series models only need the historical data of the target variable to predict its future behavior; however, they require many observations in order to produce accurate forecasts. Due to the instability of energy consumption, nonlinear intelligent prediction models have been employed, such as artificial neural network (ANN) [2-4], fuzzy regression [5, 6], and some of the hybrid models [7, 8], in order to more efficiently forecast the demand for energy. However, the prediction accuracy of the above-mentioned nonlinear models also relies on the number of training data and its representation. In developing countries, the trend of energy consumption may change rapidly over time. Therefore, only the most recent sample data are adequate for the prediction of renewable, nuclear, and total primary energy consumption. Grey prediction models, on the other hand, are appropriate when dealing with rapidly changing data because of their low data requirements.

Grey theory was first proposed by Deng [9] in 1982 and has been widely used in forecasting studies. When compared with other forecasting techniques, advantages of grey prediction model include no statistical distribution of data, small sample requirements, and high prediction accuracy [10, 11]. One of the main characteristics of grey theory is the accumulated generating operation (AGO),

and its aim is to reduce the source data to a monotonic increasing series. Some of the modified nonlinear grey hybrid models have been proposed, such as Pao et al. [12], Taguchi-grey [13], grey-Markon [14], trigonometric-grey [15], and gray-based learning model [16]. They are not only complex mathematical inference but also difficult to apply.

The nonlinear grey Bernoulli with power j model (NGBM^{*j*}) was named by Chen et al. [17] and was first mentioned in the book by Liu et al. [18]. NGBM is built based on the modification of Bernoulli differential equation in the GM model [19]. The power j in the Bernoulli differential equation can be adjusted to achieve the best prediction performances. Pao et al. [12] proposed a numerical iterative method to optimize power j in NGBM to improve model precision and the best power NGBM is named as NGBM^{op}.

The remainder of this paper is organized as follows. Section 2 outlines the GM and NGBM approaches. Section 3 presents the forecasting results and discussions. Finally, the last section concludes the paper.

2. Methodology

This section describes nonlinear grey prediction models GM (1, 1), NGBM^{*j*} (1, 1), and NGBM^{op} (1, 1). Both GM and NGBM^{op} are employed to forecast three different energy consumptions of Russia from 2009 to 2015, namely renewable, nuclear, and total primary energy consumption. NGBM^{op}'s abilities of multi-period forecasts are compared with the GM by using the out-of-sample during 2003-2008 for renewable and total energy consumption, and 2004-2009 for nuclear energy consumption, where the in-sample period is during 1997-2002 or 1998-2003.

One of the advantages of GM (1, 1), NGBM^{*j*} (1, 1), and NGBM^{op} (1, 1) grey prediction models is that they can utilize a limited amount of data to achieve accurate predictions. The value '1' in the first dimension for grey prediction models means that only one variable needs to be forecasted, and the other '1' represents the first order grey differential equation to build a grey model. Grey theory was proposed by Deng [9], the detail algorithm of GM (1, 1) was described by Pao [11], and the detail algorithms of NGBM^{*j*} (1, 1) and NGBM^{op} (1, 1) were described by Pao [12].

Based on the modification of Bernoulli differential equation in the GM model [18], the algorithm of NGBM^{*j*} (1, 1) can be summarized as follows.

Considering the non-negative time-series data:

$$v^{(0)} = [v^{(0)}(0), v^{(0)}(1), \dots, v^{(0)}(i), \dots, v^{(0)}(n)], \text{ where } n \geq 3 \quad (1)$$

NGBM^{*j*} (1, 1) is as follows:

$$v^{(0)}(k) + \alpha W^{(1)}(k) = \beta [W^{(1)}(k)]^j, \quad j \in R, \quad (2)$$

where

$$W^{(1)}(k) = 0.5[v^{(1)}(k) + v^{(1)}(k-1)], \quad k=1, 2, \dots, n \quad (3),$$

$$v^{(1)}(k) = \sum_{i=0}^k v^{(0)}(i), \quad k = 0, 1, \dots, n \quad (4)$$

$$\text{and } v^{(1)}(0) = v^{(0)}(0). \quad (5)$$

$v^{(1)}(k)$ is obtained by accumulated generating operation (AGO). The optimal value of power j in Eq. (2) is determined by the minimum mean absolute percentage error. NGBM^{*j*} is reduced to GM when $j=0$, and it is reduced to grey Verhust model when $j=2$ [18]. The parameters α and β can be estimated as

$$[\alpha, \beta]^T = [D^T D]^{-1} D^T y_n$$

where

$$D = \begin{bmatrix} -W^{(1)}(1) & [W^{(1)}(1)]^j \\ -W^{(1)}(2) & [W^{(1)}(2)]^j \\ \vdots & \vdots \\ -W^{(1)}(n) & [W^{(1)}(n)]^j \end{bmatrix} \text{ and } y_n = \begin{bmatrix} v^{(0)}(1) \\ v^{(0)}(2) \\ \vdots \\ v^{(0)}(n) \end{bmatrix}, \quad j \in R \quad (6)$$

Following is the response equation

$$\hat{v}^{(1)}(k) = \left[\left(v^{(0)}(0)^{(1-j)} - \frac{\beta}{\alpha} \right) e^{-\alpha(1-j)k} + \frac{\beta}{\alpha} \right]^{1/(1-j)}, \quad j \neq 1 \text{ and } k=0, 1, \dots \quad (7)$$

By performing inverse accumulated generating operation (IAGO) on $\hat{v}^{(1)}(k+1)$, the predicted value of $\hat{v}^{(0)}(k+1)$ is

$$\hat{v}^{(0)}(k+1) = \hat{v}^{(1)}(k+1) - \hat{v}^{(1)}(k), \quad k = 0, 1, \dots \quad (8)$$

where $\hat{v}^{(0)}(1), \hat{v}^{(0)}(2), \dots, \hat{v}^{(0)}(n)$ is called a fitted sequence, and $\hat{v}^{(0)}(n+1), \hat{v}^{(0)}(n+2), \dots$ are prediction values.

Three different statistics: RMSE, MAE, and MAPE are employed to evaluate the accuracy of the forecasts using the out-of-sample period. Lewis [20] developed a scale to evaluate forecasting performance. In this scale, if the value of MAPE is lower than 10%, it is considered highly accurate. 10-20% is good, 20-50% reasonable, while greater than 50% is considered inaccurate. The power j in NGBM can be adjusted to minimize the MAPE value using a numerical iterative method. In the next section, the iterative results will demonstrate that parameter j is efficient in improving the model precision. Also, the prediction results of NGBM with optimal power j model

(NGBM^{OP}) are compared with the results of GM (1, 1) models.

3. Forecasts and Discussion

3.1 Data analysis

In this research, we collected annual total data from EIA of renewable (R) and total energy consumption (TE) for the period from 1997 to 2008 as well as nuclear energy consumption (NE) and CO₂ emissions (CO₂) from 1997 to 2009. Real GDP data between 1997 and 2009 were obtained from World Development Indicators (WDI), and are measured in 2000 US dollars. The three different types of energy consumption are measured in quadrillion Btu (British thermal unit). CO₂ emissions are measured in Million Metric Tons (MMT), produced by the burning of fossil fuels and the manufacture of cement.

Table 1 displays the summary statistics associated with the Russian energy consumption series. The trends of these series are shown in Figs. 1-3. In Table 1, nuclear energy consumption exhibits the largest related variation (defined by coefficient of variation (CV)) while renewable shows the smallest related variation. Table 2 shows the average growth rates of the three different types of energy consumption, emissions, and real GDP. Fifteen-year (1993~2008), ten-year (1998~2008), and five-year (2003~2008) growth rates are respectively calculated to demonstrate the long-term, medium-term, and short-term growth trends. For the short-term period, the Russian compound annual growth rate (CAGR) in real GDP is 7.09%, which is almost 2.1 times higher than the world CAGR of 3.41%; nuclear boasts a CAGR of 1.76%, almost 3 times higher than the world CAGR of 0.59%. But Russia has lower CAGRs in both renewable (0.47%) and total energy use (1.31%) than the world CAGRs of respectively 3.67% and 2.98%. Russia's CAGR in emissions is 0.88%, almost 3.7 times lower than the world CAGR of 3.26%. In addition, the Russian long-term growth rates in both emissions (-0.72%) and energy use (-0.03%) are negative, while the world growth rates are positive in emission (2.30%) and energy use (2.42%). The results show that Russia is a booming market and that the government effectively conserved energy resources, controlled emission, developed clean energy, and responded to climate change in the past five years.

Table 1: Descriptive statistics for Russia from 1997 to 2008

Renewable (Quadrillion Btu)			Nuclear (Quadrillion Btu)			Total (Quadrillion Btu)		
Mean	S.D.	CV(%)	Mean	S.D.	CV(%)	Mean	S.D.	CV(%)
1.69	0.07	4.14	1.43	0.20	13.98	28.06	1.62	5.70

Table 2: Compound annual growth rates towards 2008 for each variable (in percentages)

	Russia					World				
	R	NE	TE	GDP	CO ₂	RE	NE	TE	GDP	CO ₂
15-year	-0.57	2.06	-0.03	2.79	-0.72	2.33	1.41	2.42	3.10	2.29
10-year	0.16	4.57	1.69	6.84	1.35	2.48	1.11	2.58	3.08	2.83
5-year	0.47	1.76	1.31	7.09	0.88	3.67	0.59	2.98	3.41	3.26

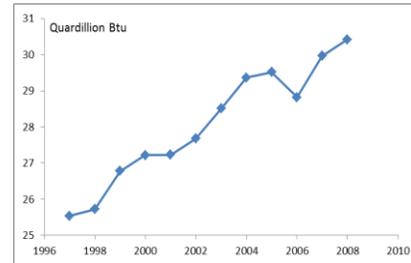


Fig. 1. Trend plot of total energy consumption from 1997 to 2008.

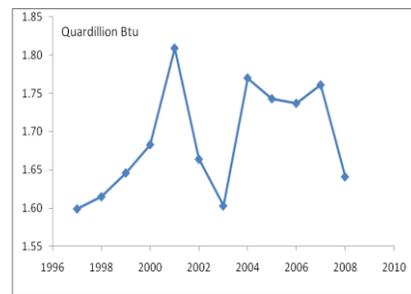


Fig. 2. Trend plot of renewable energy consumption from 1997 to 2008.

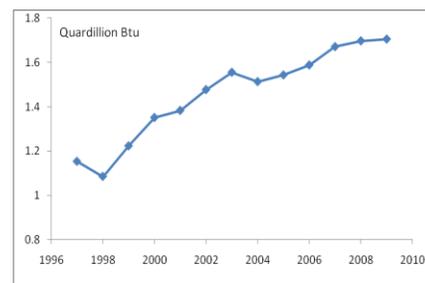


Fig. 3. Trend plot of nuclear energy consumption from 1997 to 2009.

3.2. Forecasting results

The multi-step forecasting performances of the NGBM^{OP} models are compared with the GM (1, 1) models by using out-of-sample actual data during the period 2003-2008 for

renewable and total energy consumption and 2004-2009 for nuclear energy consumption. For each variable, the GM/NGBM^{op}-k (k=6, 5 or 4) models connect six-year (GM-6, 1997-2002 or 1998-2003), five-year (GM-5, 1998-2002 or 1999-2003), and four-year (GM-4, 1999-2002 or 2000-2003) data sets as the in-sample period. The in-sample data are employed to build models, and the out-of-sample data are used to evaluate the prediction accuracy by using RMSE, MAE, and MAPE statistics. The best prediction model enjoys the lowest value of MAPE. For NGBMⁱ, the proposed numerical iterative method with MAPE value is employed to determine the optimal power *i*. Figs. 4-6 show the impact on the MAPE values in NGBM when the powers *i* are set to -0.2 to 0.2 with 0.01 increments for the three different types of energy consumption. Figures show that the proposed iteration method is an effective optimization algorithm for the power selection of NGBM. In particular, for all tested *i*, alternatives 0.07, 0.14, and 0, each for renewable, nuclear, and total energy consumption, have the lowest MAPE in NGBMⁱ (1, 1).

Three observations can be thus made, the first of which is that the best GM (1, 1) models for renewable, nuclear, and total energy consumption are GM-4, GM-5, and GM-6 with MAPE values being respectively 4.01, 13.89, and 1.39, as shown in Table 3. The parameters *a* and *b* for the best GM models are shown in Table 4. The RMSE, MAE and MAPE statistics for all of the GM-*k* (*k* = 4, 5 and 6) models are shown in Table 3. As we can see, the ranges of the MAPE values are 4.01–7.27, 13.89–18.28, and 1.39–3.42 respectively for renewable, nuclear, and total energy consumption. According to Lewis’s criteria [20], GM model presents a highly accurate forecast for renewable and total energy consumption and a good forecast for nuclear. Secondly, the best NGBM^{op} for renewable, nuclear, and total energy consumption, as shown in Table 3, are NGBM^{0.07}-6, NGBM^{0.14}-5, and NGBM⁰-6 with respective MAPE values of 2.90%, 2.20%, and 1.39% where NGBM⁰-6 is equal to GM-6. The parameters *a* and *b* for the best NGBM^{op} models are shown in Table 4. As shown in Table 3, we can see that the ranges of the MAPE values are 2.90–3.13%, 2.20–2.93%, and 1.39–1.58% respectively for renewable, nuclear, and total energy consumption, which are much lower than Lewis’s criteria, 10%. Thus, NGBM^{op} model presents a highly accurate forecast for the three different types of energy consumption. Thirdly, Figs. 4-6 show that the proposed numerical iterative method is an effective optimization algorithm for choosing optimal powers *i* in the NGBM to improve the accuracy of the model. Finally, this study finishes by using the best NGBM^{op} model to forecast the three types of energy consumption for Russia from 2009 to 2015. The forecast values, together with the

actual values, are presented in Tables 5. The prediction results show that Russia’s renewable, nuclear, and total energy consumption will grow at compound annual growth rates (CAGRs) of 1.95%, 2.44%, and 0.88% respectively over the period of 2010-2015.

The results are compared with leading research in the field of energy forecasting, e.g., Azadeh et al. [2] for Iran, Pao [21, 7] and Lee & Shih [16] for Taiwan, and Kumar and Jain [14] for India. For one-period out-of-sample forecasting, Azadeh et al. used a simulated-based ANN univariate model to forecast monthly electricity consumption in Iran, and because of ANN’s dynamic structure, the value of MAPE is lower than that of ANN. For multi-period out-of-sample forecasting, Pao [21] proposed a multivariate ECSTSP model to forecast electricity consumption in Taiwan with the value of MAPE at approximately 3.90%; Pao [7] proposed an ANN-based hybrid univariate model for energy consumption in Taiwan, and the value of MAPE is lower than 5%. Lee and Shih proposed a novel grey-based cost efficiency model (GCE) to improve short-term prediction of power generation cost for renewable energy technologies. Empirical results demonstrated that the GCE model has a highly accurate forecasting power. Additionally, Kumar and Jain applied three univariate models, namely Grey-Markov, Grey-Model with rolling mechanism, and singular spectrum analysis, in order to forecast the consumption of conventional energy (petroleum, coal, electricity, and natural gas) in India. As for the two out-of-sample forecasts (2006-2007), the MAPE values of Kumar and Jain’s models ranged from 1.6% to 3.4%. In this paper, all of the MAPE values of the best NGBM^{op} for medium-term forecasting are lower than 3.20. Therefore, NGBM^{op} shows a highly accurate predictive model for green energy systems.

Table 3: Out-of-sample comparisons between GM and NGBM models from 2003 to 2009

	GM-4	GM-5	GM-6	NGBM-4	NGBM-5	NGBM-6
Forecasts of renewable energy consumption (2003-2008)				<i>i</i> =-0.04	<i>i</i> =0.04	<i>i</i> =0.07
RMSE	0.07	0.11	0.14	0.07	0.07	0.07
MAE	0.07	0.08	0.12	0.05	0.05	0.05
MAPE(%)	4.01	5.06	7.27	3.13	2.94	2.90
Forecasts of nuclear energy consumption (2004-2009)				<i>i</i> =0.17	<i>i</i> =0.14	<i>i</i> =0.19
RMSE	0.32	0.24	0.31	0.06	0.05	0.04
MAE	0.30	0.23	0.29	0.05	0.03	0.35
MAPE(%)	18.28	13.89	17.53	2.93	2.20	2.25
Forecasts of total energy consumption (2003-2008)				<i>i</i> =-0.03	<i>i</i> =-0.03	<i>i</i> =0
RMSE	1.09	0.91	0.48	0.56	0.52	0.48
MAE	1.01	0.83	0.41	0.46	0.45	0.41
MAPE(%)	3.42	2.79	1.39	1.58	1.52	1.39

Table 4: The parameters a and b in both GM and NGBM models for energy consumptions

Parameter	Renewable energy	Nuclear energy	Total energy	Renewable energy	Nuclear energy	Total energy
	GM-4	GM-5	GM-6	NGBM ^{0.07}	NGBM ^{0.14}	NGBM ⁰
a	0.005	-0.010	-0.016	0.008	0.006	-0.016
b	1.742	1.51e+03	25.447	1.541	1.218	25.447

Table 5: Forecasts of renewable, nuclear, and total energy consumption from 2009 to 2015

Year	Renewable		Nuclear		Total energy	
	Actual	NGBM	Actual	NGBM	Actual	NGBM
2003	1.603	1.603			28.512	28.512
2004	1.770	1.813	1.513	1.513	29.370	29.106
2005	1.743	1.730	1.543	1.554	29.520	29.361
2006	1.737	1.704	1.588	1.598	28.818	29.618
2007	1.761	1.700	1.671	1.641	29.969	29.878
2008	1.641	1.709	1.697	1.684	30.426	30.139
2009		1.727	1.705	1.726		30.403
2010		1.750		1.769		30.669
2011		1.779		1.813		30.938
2012		1.811		1.858		31.209
2013		1.847		1.903		31.482
2014		1.886		1.949		31.758
2015		1.928		1.997		32.036

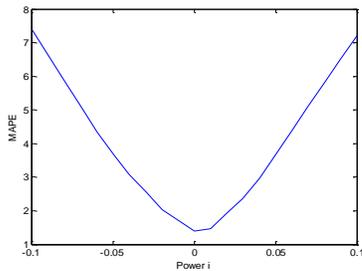


Fig. 4. MAPE values with different power i in NGBMⁱ (1, 1) for total energy consumption over the out-of-sample period of 2003-2008.

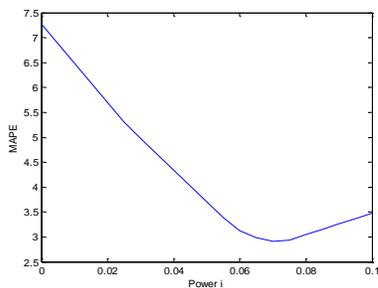


Fig. 5. MAPE values with different power i in NGBMⁱ (1, 1) for renewable energy consumption over the out-of-sample period of 2003-2008.

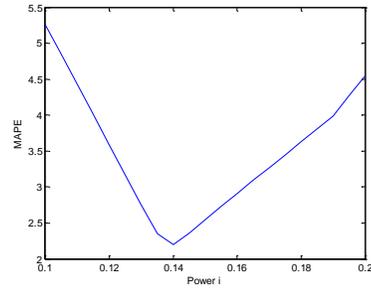


Fig. 6. MAPE values with different power i in NGBMⁱ (1, 1) for nuclear energy consumption over the out-of-sample period of 2003-2008.

4. Conclusions

Forecasts of renewable, nuclear, and total energy consumption are key requirements for a green energy system and understanding climate change in an emerging market such as Russia. This research uses recent four- to six-year historical data to construct univariate GM and NGBM^{op} models for forecasting these three indicators over the period of 2009-2015, while 1997-2003 is the in-sample period and 2004-2009 is the out-of-sample period. The multi-step forecasting ability of the best NGBM^{op} is compared with GM models over the out-of-sample period. The proposed numerical iterative method with the value of MAPE is an effective optimization algorithm for choosing optimal power of NGBM. NGBM^{0.07}-6 with a MAPE value of 2.90 for renewable and NGBM^{0.14}-5 with a MAPE value of 2.20 for nuclear are both better than GM models, whose value of MAPE is the lowest. For total energy consumption predictions, NGBM⁰-6 and GM-6 are equally good. Performance evaluation results are clear and it is shown that NGBM^{op} can be used safely for future projection of these indicators in a green energy system. Future projections have also been carried out for these indicators using NGBM^{op} for the period between 2009 and 2015. The prediction results show that Russia's renewable, nuclear, and total energy consumption will grow respectively at compound annual growth rates (CAGRs) of 1.95%, 2.44%, and 0.88% over the period of 2010-2015. The Russian government can apply these results for the dynamic adjustment of its green energy policy.

Because of the global economic uncertainty, high-tech progresses, and ever-changing domestic social structures, it is strictly recommended to revise the results every five years using NGBM^{op} to obtain more accurate outcomes. In the future, NGBM^{op} can be used to improve the accuracy of multi-step predictions of conventional or

sectoral energy consumption in other fast-growing markets to effectively develop a clean energy economy.

References

- [1] Perelet R, Pegov S, Yulkin M. Climate Change. Russia Country Paper 2007.
- [2] Catalao JPS, Pousinho HMI, Mendes VMF. Short-term wind power forecasting in Portugal by neural networks and wavelet transform. *Renewable Energy* 2011;36:1245-1251.
- [3] Kavooosi H et al. Forecast global carbon dioxide emission by use of genetic algorithm (GA). *IJCSI* 2012;9:418-427.
- [4] Pao HT. Forecasting electricity market pricing using artificial neural networks. *Energy Conversion and Management* 2007;48:907-912.
- [5] Blonbou R, Very short-term wind power forecasting with neural networks and adaptive Bayesia. *Renewable Energy* 2010;36:1118-1124.
- [6] Badar UI Islam E. Comparison of conventional and modern load forecasting techniques based on artificial intelligence and expert systems. *IJCSI* 2011;8:504-513.
- [7] Azadeh A, Khakestani M, Saberi M. A flexible fuzzy regression algorithm for forecasting oil consumption estimation. *Energy Policy* 2009;37:5567-79.
- [8] Pao HT. Forecasting energy consumption in Taiwan using hybrid nonlinear models. *Energy* 2009;34:1438-1446.
- [9] Li K, Su H. Forecasting building energy consumption with hybrid genetic algorithm-hierarchical adaptive network-based fuzzy inference system. *Energy and Buildings* 2010;42:2070-2076.
- [10] Chaabene M, Ben Ammar M. Neuro-fuzzy dynamic model with Kalman filter to forecast irradiance and temperature for solar energy systems. *Renewable Energy* 2008;33:1435-1443.
- [11] Monfared M, Rastegar H, Kojabadi HM. A new strategy for wind speed forecasting using artificial intelligent methods. *Renewable Energy* 2009;34:845-848.
- [12] Cadenas E, Rivera W. Wind speed forecasting in three different regions of Mexico, using a hybrid ARIMA-ANN model. *Renewable Energy* 2010;35:2732-2738.
- [13] Deng JL. Control problems of grey systems. *System & Control Letter* 1982;5:288-294.
- [14] Akay D, Atak M. Grey prediction with rolling mechanism for electricity demand forecasting of Turkey. *Energy* 2007;32:1670-1675.
- [15] Pao HT, Tsai CM. Modeling and forecasting the CO₂ emissions, energy consumption, and economic growth in Brazil. *Energy* 2011;36:2450-2458.
- [16] Pao HT, Fu HC, Tseng CL. Forecasting of CO₂ emissions, energy consumption and economic growth in China using an improved grey model. *Energy* 2012; 1:10.
- [17] Yao AWL, Chi SC. Analysis and design of a taguchi-grey based electricity demand predictor for energy management systems. *Energy Conversion and Management* 2004;45:1205-1217.
- [18] Kumar U, Jain VK. Time series models (Grey-Markov, GreyModel with rolling mechanism and singular spectrum analysis) to forecast energy consumption in India. *Energy* 2010;35:1709-1716.
- [19] Zhou P, Ang BW, Poh KL. A trigonometric grey prediction approach to forecasting electricity demand. *Energy* 2006;31:2839-2847.
- [20] Lee SC, Shih LH. Forecasting of electricity costs based on an enhanced gray-based learning model: A case study of renewable energy in Taiwan. *Technological Forecasting and Social Change* 2011;78(7):1242-1253.
- [21] Chen, C.I., Chen, H.L., Chen, S.P. Forecasting of foreign exchange rates of Taiwan's major trading partners by novel nonlinear grey Bernoulli model, NGBM(1,1). *Communications in Nonlinear Science and Numerical Simulation* 2008;13(6):1194-1204.
- [22] Liu SF., Dang, Y.G., Fang, Z.G. *The Theory of Grey System and its Applications*. Science Press, Beijing (in Chinese); 2004.
- [23] Chen, C.I. Application of the novel nonlinear grey Bernoulli model for forecasting unemployment rate. *Chaos Solitons & Fractals* 2008;37:278-287.
- [24] Lewis, C.D. *Industrial and Business Forecasting Method*, London: Butterworth-Heinemann; 1982.
- [25] Pao HT. Forecast of electricity consumption and economic growth in Taiwan by state space modeling. *Energy* 2009;34:1779-1791.

Hsiao-Tien Pao is a professor of the Department of Management Science at National Chiao Tung University, in Taiwan.

Hsin-Chia Fu received the B.S. degree from National Chiao-Tung University in Electrical and Communication engineering in 1972, and the M.S. and Ph.D. degrees from New Mexico State University, both in Electrical and Computer Engineering in 1975 and 1981, respectively. From 1981 to 1983 he was a Member of the Technical Staff at Bell Laboratories. From 1983 to 2012 he has been on the faculty of the Department of Computer science and Information engineering at National Chiao-Tung University, in Taiwan, ROC. Currently, he is a distinguished professor at the College of Engineering, Huaqiao University, Quanzhou, Fujian, China.

Hsiao-Cheng Yu is a professor of the Graduate Institute of Technology Management at National Chiao Tung University, in Taiwan. Currently, he is the deputy director of the National Communications Commission, in Taiwan.