Hierarchal Variable Switching Sets of Interacting Multiple Model for Tracking Maneuvering Targets in Sensor Network

Fathy Aamer, and Seham Ebrahim

Department of Information Technology, Faculty of Computer & Information Cairo University
Cairo, Egypt

Department of Computer Engineering, Modern Academy of computer engineering
Cairo, Egypt

Abstract

Tracking maneuvering targets introduce two major directions to improve the Multiple Model (MM) approach: Develop a better MM algorithm and design a better model set. The Interacting Multiple Model (IMM) estimator is a suboptimal hybrid filter that has been shown to be one of the most cost-effective hybrid state estimation schemes. The main feature of this algorithm is the ability to estimate the state of a dynamic system with several behavior modes which can “switch” from one to another. In particular, the use of too many models is performance-wise as bad as that of too few models. In this paper we show that the use of too many models is performance-wise as bad as that of too few models. To overcome this we divide the models into a small number of sets, tuning these sets during operation at the right operating set. We proposed Hierarchal Switching sets of IMM (HSIMM). The state space of the nonlinear variable is divided into sets each set has its own IMM. The connection between them is the switching algorithm which manages the activation and termination of sets. Also the re-initialization process overcomes the error accumulation due to the targets changes from one model to another. This switching can introduce a number of different models while no restriction on their number. The activation of sets depends on the threshold value of set likely hood. As the likely hood of the set is higher than threshold it is active otherwise it is minimized. The result is the weighted sum of the output of active sets. The computational time is minimum than introduced by IMM and VIMM. HSIMM introduces less error as the noise increase and there is no need for re adjustment to the Covariance as the noise increase so it is more robust against noise and introduces minimum computational time.

Keywords: *Interacting Multiple Model (IMM), Probabilistic Data Association, Sensor Network.*

1. Introduction

Multiple-model approach provides the state-of the-art solutions to many problems involving estimation, filtering, control, and/or modeling. One of the most important problems in the application of the multiple-model approach is the design of the model set used in a multiple-model algorithm. There are two types of model-set design: online and offline. Offline design is for the total model set or the initial model set in a variable-structure approach, as well as for the fixed structure approach. In a fixed-structure algorithm, the model set used cannot vary and is determined *a priori* by model-set design. In a variable-structure algorithm, the model set in effect at any time is determined by an adaptation process, known as *model-set adaptation*, which may be viewed as an online (real-time) design process and will depend on the total model set determined *a priori* if such a set exists. In this paper we study the IMM with a large number of operating modes and introduce its performance for different modes changes. Then we replace this IMM with another structure of IMM. The structure includes sets of IMM. Each set includes part of these operating modes. At initialization all the sets are active but the right set which introduce lower innovation will be active while the others will be switched off When the system change to a different active set the diverging of the active set will cause system initialization and activation to all sets and start to tune to the right set. This algorithm overcome the problem of large number of modes in IMM, activate only the right set not all sets which introduce less computation complexity, and allow variation of time step to large values to reduce energy consumption in Sensor Network while tracking.

In section 2 the related work is presented. We introduce IMM in section 3. Variable structure IMM is in section 4 while the HSIMM in section 5. The Results of IMM of The First Tracking Problem is in section 6.The design of HSIMM is introduced in section 7 while its results of the first tracking problem in section 8. The second tracking dynamics and its results are introduced in section 9, finally section 10 presents conclusion and future work.

2. The Related Work

The IMM was introduced in [1] which summarized the state-of-the-art of the IMM and its variants. They discuss and compare the base-line IMM algorithm with variable-structure variants, multiple sensor variants, correlated noise variants, glint-noise influenced variants, and others to know more about IMM. But it is shown theoretically that the use of too many models is performance-wise as bad as that of too few models. In [7] they introduced difficulties of excess of measure while [8] introduced the difficulties of using IMM in Radar system. In [2] they introduced limitations of the fixed structure algorithm. Then presented theoretical results pertaining to the two ways of overcoming these limitations. select/construct a better set of models and/or use a variable set of models. The new approach was illustrated in non stationary noise identification problem. In our previous work [12] considering Extended Kalman filter and IMM in the same tracking problem to introduce the accuracy and time delay of the two tracking algorithm. In this paper structure of IMM sets are introduced to overcome limited number of sets and nonlinearity of target motion model. The structure includes number of IMM (set of modes) working separately. But the right set will be considered while the others will be switched off.

3. The Interacting Multiple Model

The system is described by the model:

$$x(k+1) = F(k, m(k+1))x(k) + G(k, m(k+1)) * u(k, m(k+1)),\ldots \quad (1)$$

$$z(k) = H(k)x(k) + w(k),\ldots \quad (2)$$

Where $x \in \Re^{n_x}$ is the system state vector, $z \in \Re^{n_z}$ is the measurement vector, $u \in \Re^{n_u}$ and $w \in \Re^{n_z}$ are mutually uncorrelated, white zero mean Gaussian noises with coveriances Q_u and Rw respectively. The parameter m_k presents the current system mode. F is the system dynamic matrix, and H is the measuring matrix.

Because the accurate system model is unknown, the system is described by a number of models. The event that the i^{th} model m_i is actual at time k is denoted as $M_i(k)=\{m(k)=m_i\}$.

It is assumed that the system model sequence is a Markov chain with transition probabilities

$$P\{m_j(k+1) \mid m_i(k)\} = P_{r_{ij}}(k)$$

$$\text{where } \sum_{j=1}^{r} P_{r_{ij}}(k) = 1, \qquad i = 1,2.....,r$$

Designing model set is the major element affects the performance of the IMM. These models represent different mode of operation of the system. If the system operating mode is fare from these models it doesn't converge. Or if we add models near to each other in parameters the estimated state may converge to the wrong model and also models probabilities after converging to the wrong model cause system divergence.

We introduce two models one linear motion and one coordinated turn model for quick maneuver detection.

1) Constant Velocity Model: This model is the most commonly used. The target is assumed to move with a constant velocity. For notational simplicity, $x_t \cong \{x_t, \dot{x}_t, y_t, \dot{y}_t\}$ refers to the state (coordinates and the velocities) of a single target following this motion model and u_t is the corresponding motion noise.

$$x_{t+1} = \begin{Bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{Bmatrix} x_t + \begin{Bmatrix} \frac{\Delta T^2}{2} & 0 \\ \Delta T & 0 \\ 0 & \frac{\Delta T^2}{2} \\ 0 & \Delta T \end{Bmatrix} u_t \quad (3)$$

where $u_t \sim N(0, diag(\pm_x^2, \pm_y^2))$.

2) Coordinated Turn Rate Model: This model assumes that the target moves with a constant speed (norm of the velocity vector) and a constant known turn rate μ. Again, we denote x_t as the state of a single target from this class and u_t as the corresponding motion noise. To introduce the case of a target moving with varying μ change from 0.2 to 1.8 we divide the IMM into 10 modes, one linear and other 9 modes at different μ

$$x_{t+1} = \begin{pmatrix} 1 & \frac{\sin \mu \Delta T}{\mu} & 0 & -\frac{1-\cos \mu \Delta T}{\mu} \\ 0 & \cos \mu \Delta T & 0 & -\sin \mu \Delta T \\ 0 & \frac{1-\cos \mu \Delta T}{\mu} & 1 & \frac{\sin \mu \Delta T}{\mu} \\ 0 & \sin \mu \Delta T & 0 & \cos \mu \Delta T \end{pmatrix} x_t + \begin{pmatrix} \frac{\Delta T^2}{2} & 0 \\ \Delta T & 0 \\ 0 & \frac{\Delta T^2}{2} \\ 0 & \Delta T \end{pmatrix} u_t \quad (4)$$

Where u_t has the same Gaussian distribution as in (3).

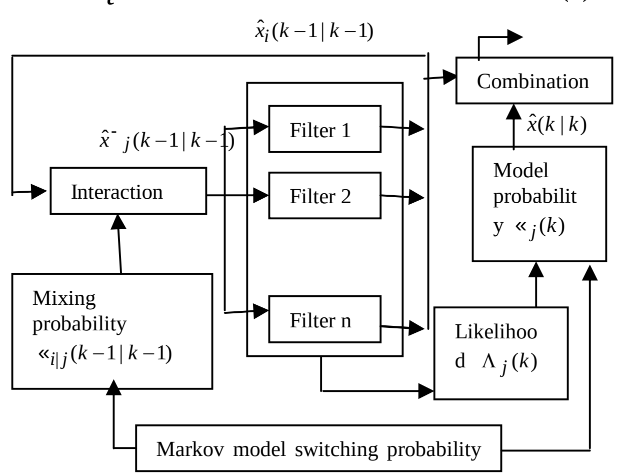

Fig. 1 Scheme of IMM algorithm

A Markov transition matrix is used to specify the probability that the target is in one of the modes of operation. The model probabilities are updated at each new measurement, and the resulting weighting factors are used in calculating the state. One cycle of a practical IMM algorithm consists of the following steps which in [3],[4],[9]

One Cycle of the IMM Estimator

1. Model-conditioned re initialization (for $i = 1, 2, \ldots, M$):

Predicted model probability: $\hat{\mu}^{(i)}_{k|k-1} \overset{\Delta}{=} P\{m_k^{(i)} \mid z^{k-1}\} = \sum_j \pi_{ji} \mu^{(j)}_{k-1}$

Mixing weight: $\mu^{j|i}_{k-1} \overset{\Delta}{=} P\{m^{(j)}_{k-1} \mid m_k^{(i)}, z^{k-1}\} = \pi_{ji} \mu^{(j)}_{k-1} / \mu^{(i)}_{k|k-1}$

Mixing estimate: $\bar{x}^{(i)}_{k-1|k-1} \overset{\Delta}{=} E[x_{k-1} \mid m_k^{(i)}, z^{k-1}] = \sum_j \hat{x}^{(j)}_{k-1|k-1} \mu^{j|i}_{k-1}$

Mixing covariance: $\bar{P}^{(i)}_{k-1|k-1} = \sum_j [P^{(j)}_{k-1|k-1} + (\bar{x}^{(i)}_{k-1|k-1} - \hat{x}^{(j)}_{k-1|k-1})(\bar{x}^{(i)}_{k-1|k-1} - \hat{x}^{(j)}_{k-1|k-1})'] \mu^{j|i}_{k-1}$

2. Model-conditioned filtering (for $i = 1, 2, \ldots M$):

Predicted state: $\hat{x}^{(i)}_{k|k-1} = F^{(i)}_{k-1} \bar{x}^{(i)}_{k-1|k-1} + G^{(i)}_{k-1} \bar{w}^{(i)}_{k-1}$

Predicted covariance: $P^{(i)}_{k|k-1} = F^{(i)}_{k-1} \bar{P}^{(i)}_{k-1|k-1} (F^{(i)}_{k-1})' + G^{(i)}_{k-1} Q^{(i)}_{k-1} (G^{(i)}_{k-1})'$

Measurement residual: $\tilde{z}^{(i)}_k = z_k - H^{(i)}_k \hat{x}^{(i)}_{k|k-1} - \bar{v}^{(i)}_k$

Residual covariance: $S^{(i)} = H^{(i)}_k P^{(i)}_{k|k-1} (H^{(i)}_k)' + R^{(i)}_k$

Filter gain: $K^{(i)}_k = P^{(i)}_{k|k-1} (H^{(i)}_k)' (S^{(i)}_k)^{-1}$

Updated state: $\hat{x}^{(i)}_{k|k} = x^{(i)}_{k|k-1} + k^{(i)}_k \tilde{z}^{(i)}_k$

Updated covariance: $P^{i}_{k|k} = P^{(i)}_{k|k-1} - k^{(i)}_k S^{(i)}_k (k^{(i)}_k)'$

3. Model probability update (for $i = 1, 2, \ldots M$):

Model likelihood: $L^{(i)}_k \overset{\Delta}{=} p[\tilde{z}^{(i)}_k \mid m^{(i)}_k, z^{k-1}] \overset{assume}{=} \dfrac{\exp[-(1/2)(z^{(i)}_k)'(S^{(i)}_k)^{-1} \tilde{z}^{(i)}_k]}{\left|2\pi S^{(i)}_k\right|^{1/2}}$

Model probability: $\mu^{(i)}_k \overset{\Delta}{=} P[m^{(i)}_k / z^k] = \dfrac{\mu^{(i)}_{k|k-1} L^{(i)}_k}{\sum_j \hat{\mu}^{(j)}_{k|k-1} L^{(j)}_k}$

4. Estimate fusion: $\hat{x}_{k|k} \overset{\Delta}{=} E[x_k \mid z^k] = \sum_i \hat{x}^{(i)}_{k|k} \mu^{(i)}_k$

Overall estimate: $\hat{x}_{k|k} \overset{\Delta}{=} E[x_k \mid z^k] = \sum_i \hat{x}^{(i)}_{k|k} \mu^{(i)}_k$

Overall covariance: $P_{k|k} = \sum_i \left[P^{(i)}_{k|k} + (\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})(\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})'\right] \mu^{(i)}_k$

4. Variable Structure Multiple Model Estimation

General speaking, a fixed structure MM (FSMM) algorithm is one with a fixed set of models while a variable structure MM (VSMM) algorithm is one with a variable set of models. The set of models used by MM algorithm at time k is denote by M_k and the total set of models is denoted **M.** As M is the union of all M_k's. The MM algorithm is said to have a fixed structure if the model set M_k used is fixed over time (i.e. $M_k = \mathbf{M}$; $\forall k$). Otherwise it is said to have a variable structure.

VSIMM Recursion

1. Model-set conditioned (re)initialization [$\forall m_i \in M_k$]:

Predicted model probability:

$\hat{\mu}^{(i)}_{k|k-1} \overset{\Delta}{=} P\{m_k^{(i)} \mid M_k, M_{k-1}, z^{k-1}\} = \sum_{m_j \in M_k} \pi_{ji} \mu^{(j)}_{k-1}$

Mixing weight: $\mu^{j|i}_{k-1} \overset{\Delta}{=} P\{m^{(j)}_{k-1} \mid m_k^{(i)}, M_{k-1}, z^{k-1}\} = \pi_{ji} \mu^{(j)}_{k-1} / \mu^{(i)}_{k|k-1}$

Mixing estimate: $\bar{x}^{(i)}_{k-1|k-1} \overset{\Delta}{=} E[x_{k-1} \mid m_k^{(i)}, z^{k-1}] = \sum_{m_j \in M_k} \hat{x}^{(j)}_{k-1|k-1} \mu^{j|i}_{k-1}$

Mixing covariance:

$\bar{P}^{(i)}_{k-1|k-1} = \sum_{m_j \in M_k} [P^{(j)}_{k-1|k-1} + (\bar{x}^{(i)}_{k-1|k-1} - \hat{x}^{(j)}_{k-1|k-1})(\bar{x}^{(i)}_{k-1|k-1} - \hat{x}^{(j)}_{k-1|k-1})'] \mu^{j|i}_{k-1}$

2. Model-conditioned filtering [$\forall m_i \in M_k$]:

Predicted state: $\hat{x}^{(i)}_{k|k-1} \overset{\Delta}{=} E[x_k \mid m_k^{(i)}, M_{k-1}, z^{k-1}] = F^{(i)}_{k-1} \bar{x}^{(i)}_{k-1|k-1} + G^{(i)}_{k-1} \bar{w}^{(i)}_{k-1}$

Predicted covariance: $P^{(i)}_{k|k-1} = F^{(i)}_{k-1} \bar{P}^{(i)}_{k-1|k-1} (F^{(i)}_{k-1})' + G^{(i)}_{k-1} Q^{(i)}_{k-1} (G^{(i)}_{k-1})'$

Measurement residual:

$\tilde{z}^{(i)}_k = z_k - E[z_k \mid m^{(i)}_k, M_{k-1}, z_{k-1}] = z_k - H^{(i)}_k \hat{x}^{(i)}_{k|k-1} - \bar{v}^{(i)}_k$

Residual covariance: $S^{(i)} = H^{(i)}_k P^{(i)}_{k|k-1} (H^{(i)}_k)' + R^{(i)}_k$

Filter gain: $K^{(i)}_k = P^{(i)}_{k|k-1} (H^{(i)}_k)' (S^{(i)}_k)^{-1}$

Updated state: $\hat{x}^{(i)}_{k|k} \overset{\Delta}{=} E[x_k \mid m^{i}_k, M_{k-1}, z^k] = x^{(i)}_{k|k-1} + k^{(i)}_k \tilde{z}^{(i)}_k$

Updated covariance: $P^{i}_{k|k} = P^{(i)}_{k|k-1} - k^{(i)}_k S^{(i)}_k (k^{(i)}_k)'$

3. Model probability update (for i = 1; 2; : : : ;M):

Model likelihood:

$L^{(i)}_k \overset{\Delta}{=} p[\tilde{z}^{(i)}_k \mid m^{(i)}_k, z^{k-1}] \overset{assume}{=} \dfrac{\exp[-(1/2)(z^{(i)}_k)'(S^{(i)}_k)^{-1} \tilde{z}^{(i)}_k]}{\left|2\pi S^{(i)}_k\right|^{1/2}}$

Model probability: $\mu^{(i)}_k \overset{\Delta}{=} P[m^{(i)}_k / z^k] = \dfrac{\mu^{(i)}_{k|k-1} L^{(i)}_k}{\sum_j \hat{\mu}^{(j)}_{k|k-1} L^{(j)}_k}$

4. Estimate fusion: $\hat{x}_{k|k} \overset{\Delta}{=} E[x_k \mid z^k] = \sum_i \hat{x}^{(i)}_{k|k} \mu^{(i)}_k$

Overall estimate: $\hat{x}_{k|k} \overset{\Delta}{=} E[x_k \mid z^k] = \sum_i \hat{x}^{(i)}_{k|k} \mu^{(i)}_k$

Overall covariance: $P_{k|k} = \sum_i \left[P^{(i)}_{k|k} + (\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})(\hat{x}_{k|k} - \hat{x}^{(i)}_{k|k})'\right] \mu^{(i)}_k$

As the outcome of the advances during the past three decades, the state-of-the art FSMM estimators usually perform quite well for problems that can be handled by the use of a small set of models. Consequently, they have found a great success in solving many state estimation problems compounded with structural or parametric uncertainty, particularly in target tracking. However, when they are applied to solve many real-world problems (e.g., many practical target-tracking problems), it is often the case that the use of only a few models is not good enough. Although further development is certainly possible, the FSMM estimation techniques have arrived at such a stage

that great improvement can no longer be expected. This perception is based on an understanding of the fundamental limitations of the FSMM approach.
These limitations stem from the following facts:

- It assumes fundamentally that the system mode at any time can be represented (with a sufficient accuracy) by one of a fixed set of models that can be determined in advance.
- The set of possible system modes is not fixed. It depends on the hybrid state of the system at the previous time.
- As shown in [34],[38], use of more models in an FSMM estimator does not necessarily improve performance; in fact, the performance will deteriorate if too many models are used.
- It cannot incorporate certain types of a priori information.
- Clearly, the amount of computational resource required by an FSMM estimator increases dramatically with the number of models used.

5. Hierarchal Switching sets of IMM

The variable IMM increase the accuracy of the estimated position but still doesn't solve the problem of increasing number of models to estimate a wide range of variation. It works as IMM but takes only the results of models with relativity higher model probability.
In our proposed algorithm we mix the advantage of small number of models of IMM and variation of its activation over a wide range. The space plane of the nonlinear variable (e.g. acceleration or turning angle) is divided into sets. Each set has its own IMM with its transition matrix and model probability as if it stands alone. The likelihood of each set is calculated according to its introduced innovation. The output of sets is calculated as in VIMM. A threshold value to the introduced innovation cause the switching between the sets. The right set will be on while the other will be off which reduced the computation time. As the right set deviates from being the right one, all other sets will start to work while this set will be off. The overall computation time is less than that of including all sets as in IMM and VIMM. The accuracy isn't as IMM or VIMM due to the activation of off sets. Overall the Hierarchal Variable Switching of IMM overcomes the limitation of increasing number of models of IMM with high stability against measuring noise and also reduces the computation time.

5.1 Model-Set Probability and Likelihood

As in VIMM the *marginal* likelihood of a model-set M_j at time k is the sum of the predicted probabilities times the marginal likelihoods, both of all the *models* in M_j

$$L_k^{M_j} \overset{\Delta}{=} p[\tilde{z}_k \mid s \in M_j, z^{k-1}] = \sum_{m_i \in M_j} \underbrace{P[\tilde{z}_k \mid s = m_i, z^{k-1}]}_{\text{Model marginal likelihood}} \; \underbrace{P\{s = m_i \mid s \in M_j, z^{k-1}\}}_{\text{Predicted model Probability}} \quad (5)$$

Where $\tilde{z}$ the measurement residual and s is is the mode in effect during the time period over which the test is performed. Note that s has to be assumed constant because a hypothesis cannot be time variant. The *joint* likelihood of the model-set

M_j is defined as $L_{M_j}^k \overset{\Delta}{=} p\{\tilde{z} \mid s \in M_j]$. The (posterior) probability that the true mode is in a model-set M_j is defined as

$$M_k^{M_j} \overset{\Delta}{=} P\{s \in M_j \mid s \in \mathrm{M}_k, \tilde{z}\} = \sum_{m_j \in M_j} P\{s = m_i \mid s \in \mathrm{M}_k, \tilde{z}\} \quad (6)$$

Which is the sum of the probabilities of all models in M_j, where M_k is the total model-set in effect at time k, which includes M_j as a subset and is problem dependent. The model probability $P\{s = m_i \mid s \in \mathrm{M}_k, \tilde{z}\}$ for each model m_i is typically available from an IMM estimator.

5.2 Initialization of New Models and Filters

The key to the optimal initialization of new models and filters is the concept of state dependency of the system mode set. It simply states that given the current mode (and base state), the set of possible modes at the next time is a subset of the mode space, which is determined by the (Markovian) mode transition law. The optimal assignment of the initial probability to a new model accounts only for the probabilities of those models that may jump (switch) to this new model, and the optimal initial state estimate for a filter based on a (new or old) model is determined only from the estimates (and the probabilities) of the filters based on those models that may jump to the model.

Specifically, the optimal initialization of a filter based on a new (or old) model m_n can be done as follows. When calculating $E[x_k \mid m_k^{(n)}; M^{k-1}, \tilde{z}]$, only the previous estimates $\hat{x}_{k-1|k-1}^{(l)}$ based on models in the set E_n should be used, where E_n is the set of models in M_{k-1} that are allowed to switch to m_n:

$$E_n = \{m_l : m_l \in M_{k-1}, ®_{l,n} \neq 0\} \quad (7)$$

Specifically, the initial estimate for time k cycle of the filter based on model m_n can be obtained as

$$\bar{x}_{k-1|k-1} = E\left[x_{k-1} \mid m_k^{(n)}, M^{k-1}, z^{k-1}\right]$$
$$= \sum_{m_l \in E_n} E\left[x_{k-1} \mid m_{k-1}^{(l)}, M^{k-2}, z^{k-1}\right] P\left\{m_{k-1}^{(l)} \mid m_k^{(n)}, M^{k-1}\right\} \quad (8)$$
$$= \sum_{m_l \in E_n} \hat{x}_{k-1|k-1}^{(l)} « _{k-1}^{l|n}$$

Where

$$«_{k-1}^{l|n} = P\left\{m_{k-1}^{(l)} \mid m_k^{(n)}, M^{k-1}\right\} = \frac{®_{l,n} «_{k-1}^{(l)}}{\sum_{m_i \in E_n} ®_{in} «_{k-1}^{(l)}} \quad (9)$$

5.3 Adding and Removing Sets

Perform N model-set sequential likelihood ratio test. $(H_0 : s \in M_0 \; vs \; H_1 : s \in M_1)$,.... $(H_{N-1} : s \in M_{N-1} \; vs \; H_N \; s \in M_N)$

These tests are implemented by using thresholds $B = \frac{\sim}{1-}$ and A=-∞ . This step ends when only one of the hypotheses H_1, H_2;……,H_N remains. Specifically:

- Reject set that includes M_i for which $\Lambda_i^k \overset{\Delta}{=} L_M^k / L_{M_i}^k \geq B$
- Continue to the next time cycle to test for the remaining pairs with one more measurement until only one of the hypotheses H_1,H_2,…..,H_N, say H_j , is not rejected.

5.4 Re-initialization of the off sets

The likelihood functions for filter j is as follows:

$$\Lambda_j(k) = N\left[\tilde{z}_j(k); 0; S_j(k)\right]$$
$$= \left|2®S_j(k)\right|^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2}\tilde{z}_j^T(k) S_j^{-1}(k) \tilde{z}_j(k)\right] \quad (10)$$

Where $\tilde{z}_j(k) = z(k) - \bar{z}(k|k-1)$ is the innovation for filter j and $S_j(k)$ is the covariance matrix.

Switching between active sets as

$$\Lambda_j(k) - \Lambda_j(k-1) > \sim \quad (11)$$

The model set of M_j is change to off state and initiate the other two sets.

One Cycle of Switching sets of IMM

Start one cycle for each set of IMM
$\Lambda(k)_i = \max\left(\Lambda(k)_{i,j}\right)$ *where* $j = 1,2,....$*and* $i = 1,2,3$
For i=1:3
If $\Lambda(k)_i >$ *Set I change to off state*
Else active
End.
If $\Lambda(k)_i - \Lambda(k-1)_i > \sim$
Change the state to off and activate the other sets
Else set is active.
End.
End.
Compare all $\Lambda_i(k)$ *the smallest* $\Lambda_i(k)$ *takes its highest mode*
I probability to be the set probability
Set probability of the other two sets=1- probability of minimum $\Lambda_i(k)$
If the remaining sets are active
Distribute this value between them according
to their maximum model probability.
Else
Set the probability of the off sets to zero
End.
For active set which has the maximum probability we take its output
$\hat{x}_{k|k} \overset{\Delta}{=} E[x_k \mid z^k] = \sum_i \hat{x}_{k|k}^{(i)} «_k^{(i)}$

6 The Results of IMM of The First Tracking Problem

We tested our model using Matlab 2010Ra. intel core 2 duo., under windows vista environment. The following results is for 10 modes one for linear motion and the others are at different ω [0.2,0.4,0.6,0.8,0.9,1.1,1.4,1.6,1.8], Q=diagonal(0.5^2) R=diagonal(100). The results are the average of 200 run. The target trajectory described in

Table1: target trajectory

scenario	First trajectory	Second trajectory
0<k<90	ω=1.4	ω=1.4
90<k<150	ω =0.2	ω=1.6
150<k<200	linear	linear
200<k<300	ω=1.9	ω=1

The transition matrix has a diagonal of 0.82 while all models start with equal model probability

Table 2: The results of the first target trajectory

IMM						
RMSE x	RMSEy	RMSEv x	RMXEvy	Value	ΔT	Exe.time
0.6174 2.9391	0.6039 2.711	0.3242 1.416	0.2970 1.528	Mean Max	2.3	0.01443
0.6275 2.041	0.691 2.9366	0.4422 1.504	0.3774 1.6223	Mean Max	2.5	0.01443
0.7423 3.3473	0.6081 2.4812	0.2973 0.9727	0.3163 1.4287	Mean Max	2.9	0.014402
1.0326 4.9509	1.0102 4.509	0.3486 1.2098	0.4908 1.2983	Mean Max	3.3	0.014498

Table3: the results of the second target trajectory

IMM						
RMSE x	RMSEy	RMSEv x	RMXEvy	Value	ΔT	Exe time
0.6174 2.9391	0.6039 2.711	0.3242 1.416	0.2970 1.528	Mean Max	2.3	0.01443
0.6275 2.041	0.691 2.9366	0.4422 1.504	0.3774 1.6223	Mean Max	2.5	0.01443
0.7423 3.3473	0.6081 2.4812	0.2973 0.9727	0.3163 1.4287	Mean Max	2.9	0.01440
1.0326 4.9509	1.0102 4.509	0.3486 1.2098	0.4908 1.2983	Mean Max	3.3	0.01449 8

7. Hierarchal Switching of IMM

In the above examples we change models with ω near to each other while the change of linear model can take place at any k . the sudden jump of ω to far values cause system to diverge. Also in this model we choose ω that can introduce good results with each other not all values of them cause system converge or good tracking. Also Not wide range of the time step variation is available.

Two overcome these limitation we introduce a variable set of models. We take the advantage of good tracking of small sets of IMM and divide our structure into three sets

The first set includes $\omega_{1=}$[0.2 0.4 0.6] .The second set includes $\omega_{2=}$[0.8 0.9 1.1]

The third set includes linear model with $\omega_{3=}$[1.4 1.6 1.8] .The variable structure of IMM algorithm is shown in figure 2

The transition matrix of the first two set and model probability as follow

$$\Pr = \begin{bmatrix} 0.98 & 0.01 & 0.01 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.01 & 0.98 \end{bmatrix} \qquad « = \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}$$

The transition matrix of the third set has 0.97 diagonal and equal model probability

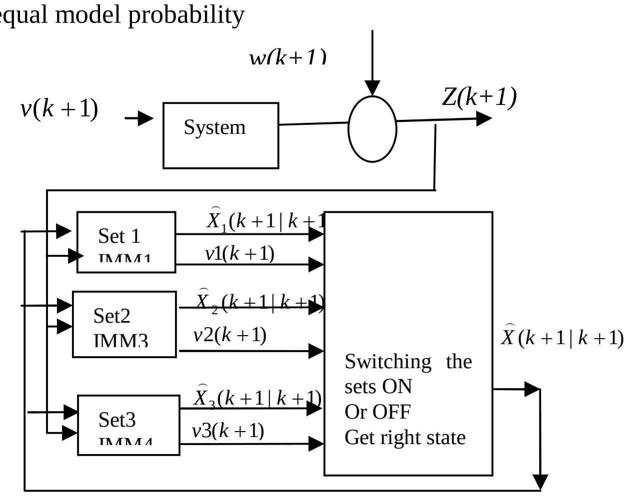

Fig.2 The Hierarchal structure of IMM for the three sets

8. The results of Hierarchal Structure of IMM for The First Tracking problem

We tested our model using Matlab Ra2010 on intel core 2 duo., under windows vista environment. The following results is for 10 modes one for linear motion and the others are at different ω [0.2,0.4,0.6,0.8,0.9,1.1,1.4,1.6,1.8], Q=diagonal(0.5^2) R=diagonal(100). The results are listed in table 4. The system characterized by unknown changeable structure as in the previous first example.

Table:4 The results of the first tracking problem

IMM						
RMSEx	RMSEy	RMSEvx	RMXEvy	value	ΔT	Exe. time
2.2637 18.505 3	1.1498 5.039	0.6093 3.6603	0.4307 2.5497	Mean Max	2.3	0.0066
2.6597 7.2371	2.4558 7.3775	1.0951 4.0049	0.9815 4.078	Mean Max	2.5	0.0066
0.6341 5.1443	0.6489 5.034	0.5999 1.8247	0.6096 1.5653	Mean Max	2.9	0.0067
2.5489 7.929	2.7573 7.603	0.8608 3.2303	0.8229 3.0638	Mean Max	3.3	0.0065
3.4279 14.176 2	3.2624 13.3039	1.1856 3.9484	1.2166 3.9018	Mean Max	4	0.0067
2.4515 8.2294	2.6506 9.7918	0.8125 3.1517	0.9002 5.5053	Mean Max	5	0.0065

If we change ω to ω_1=[7/dt 8/dt 9/dt] ω_2=[4/dt 5/dt 6/dt] and ω_3 =[1/dt 2/dt 3/dt] with linear model. These values include most of the turning angles that have good separation between modes of operation. The other values included in these modes. The RMSE for the same parameters of the IMM are listed in table 5 while the results of RMSE if we choose R=I 25, are listed in table 6

Table 5: The result of the first tracking problem

HSIMM						
RMSEx	RMSEy	RMSEvx	RMSEvy	value	ΔT	Exe time
17.6978 1.8674	18.2421 2.1699	2.2899 0.5395	2.6145 0.5922	Max Mean	2.3	0.0071
7.6344 0.872	12.4231 1.1034	3.0548 0.6205	1.9364 0.4502	Max Mean	2.5	0.0068
10.5143 0.9774	15.9724 1.2805	2.2557 0.4727	1.8583 0.4726	Max Mean	2.9	0.007
15.8625 1.1525	11.5765 1.0396	2.3474 0.4897	2.4372 0.4882	Max Mean	3.3	0.007
13.3789 1.0023	7.4274 0.9793	2.8286 0.9544	2.2394 0.8300	Max Mean	4	0.0067
11.5338 1.6121	14.1307 1.3044	1.814 0.6895	3.4595 0.8429	Max Mean	5	0.0067

Table 6: The result of the Other sets for the same target trajectory

HSIMM						
RMSE x	RMSEy	RMSEv x	RMSEvy	Value	ΔT	Exe.time
4.0811 0 .4532	7.2529 0.7541	1.4272 0.4739	1.4737 0 .4404	Max Mean	2.3	0.0067
4.893 0 .7274	6.7546 0.5493	1.8966 0.4879	1.5485 0.4558	Max Mean	2.5	0.0068
5.1443 0.6341	5.034 0.6489	1.8247 0.5999	1.5653 0.6096	Max Mean	2.9	0.0067
7.3538 0.6939	4.0377 0.7537	2.7559 0.7178	2.2064 0.6819	Max Mean	3.3	0.0068
4.773 0.7348	6.9591 1.0398	3.2547 0.6877	3.3066 0.6414	Max Mean	4	0.0061
9.954 1.5052	5.7863 1.0657	2.9827 1.0743	3.3516 1.0657	Max Mean	5	0.0068

9. Results of The second Tracking Problem

The target measurement model

$$x(k+1) = Fx(k) + G[a(k) + w(k)] \quad (12)$$

$$z(k+1) = Hx(k+1) + v(K+1); k = 0,1,2,.... \quad (13)$$

Where $x = (x, v_x, y, v_y)'$ denotes the targets state $a = (a_x, a_y)'$ is the acceleration, $w \sim N[0,Q]$ is the acceleration process noise, $z = (z_x, z_y)$ is the measurement, $v \sim N[0,R]$ is the random measurement error and

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad G = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}$$

The unknown true acceleration is assumed piecewise constant, varying over a given continuous planar region A^c. In the MM framework, we consider a generic finite set (grid) of acceleration values: $A^{(r)} \stackrel{\Delta}{=} \{a^{(i)} \in A^c : i = 1,2,...r\}$ which defines the total model set. We approximate the evolution of the true acceleration over the quantized set $A^{(r)}$ via a Markov chain model, that is, $a_k \in A^{(r)}$ with given $P\{a_0 = a^{(i)}\} = P_i$ and $P\{a_k = a^{(i)} \mid a_{k-1} = a^{(i)}\} = ®_{ij}$ for $i, j = 1,2,....r$.

Consider the following target-tracking example, adopted from [8],[18],[17],[19]. A target moves in the horizontal plane that may have piecewise-constant acceleration with a maximum value of 4g (40m/s2) in any direction. Assume that the following set of 13 time-invariant models, characterized by the expected acceleration vector a, is used:

$$\begin{Bmatrix} a_1 = 20[0,0] & a_2 = 20[1,0] & a_3 = 20[0,1] & a_4 = 20[-1,1] \\ a_5 = 20[0,-1] & a_6 = 20[1,1] & a_7 = 20[-1,1] & a_8 = 20[-1,-1] \\ a_9 = 20[1,-1] & a_{10} = 20[2,0] & a_{11} = 20[0,2] & a_{12} = 20[-2,0] \\ a_{13} = 20[0,-2] & & & \end{Bmatrix}$$

The transition relations among models are easily understood in terms of the directed graph (i.e., digraph) representation of an MM, introduced in [13], [15], [14], [16]. The topology of model set $A^{(13)}$ is depicted in Figure 3. Each model is viewed as a point in the mode (acceleration) space. An arrow from one model to another indicates a legitimate model switch (self-loops are omitted) with non-zero probability.

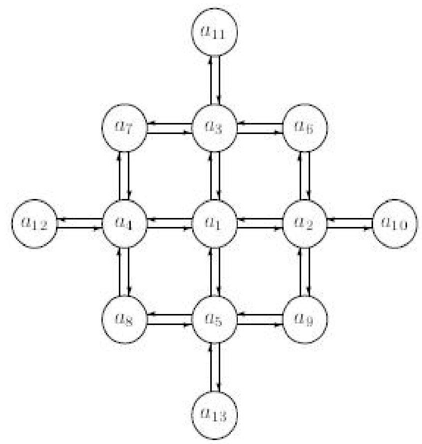

Fig. 3 Digraph representation of 13-model set

8.1 Design of VIMM and HSIMM

For both VIMM and HSIMM we divide the sets into three sets. $M_1 = \{a_2, a_3, a_{11}, a_6, a_{10}\}$, $M_2 = \{a_9, a_5, a_{13}, a_8\}$ and $M_3 = \{a_1, a_4, a_7, a_{12}\}$

The probability transition matrix of IMM13 and VIMM has diagonal of 0,8

The model probability is

$$« = [0.08 \quad 0.08 \quad 0.08 \quad 0.076 \quad 0.076 \quad 0.076 \quad 0.076 \quad 0.076 \quad 0.076 \quad 0.076 \quad 0.076 \quad 0.076 \quad 0.076]$$

For HVSIMM we have three sets each set has its own transition matrix and model probability as follow

Set1

$$® = \begin{bmatrix} 0.8 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.8 & 0.04 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.8 & 0.04 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.8 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.8 \end{bmatrix} \qquad « = [0.2 \quad 0.2 \quad 0.2 \quad 0.2 \quad 0.2]$$

Set2 and Set3

IJCSI International Journal of Computer Science Issues, Vol. 10, Issue 1, No 2, January 2013
ISSN (Online): 1694-0814 | ISSN (Print): 1694-0784
www.IJCSI.org

$$® = \begin{bmatrix} 0.97 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.97 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.97 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.97 \end{bmatrix} \qquad « = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$

The sets probability is initialized by $«_{Set} = \begin{bmatrix} 0.4 & 0.3 & 0.3 \end{bmatrix}$. It is changed according to the maximum model probability of the model of sets. The active set takes higher value while the other is distributed according to their higher model probability.

8.2 Performance Evaluation

Test scenarios

The performances of the IMM, VIMM and HSIMM tracking algorithms were investigated first over a large number of deterministic maneuver scenarios with fixed acceleration sequences. Deterministic scenarios serve to evaluate algorithms' peak errors, steady-state errors and response times. We present two of them, referred to as DS1 and DS2, in the sequel. Their acceleration values are given in Table 7

Deterministic Scenarios' Parameters

The other parameters for both scenarios are $T = 1s$; $Q = O$; $R = 1250I$; $x0 = [8000; 25; 8000; 200]$. Note that while the acceleration values in DS1 are relatively close to the fixed grid points of IMM13, in DS2 they are deliberately chosen far apart from the grid points. As such, for the fixed structure estimator IMM13 the scenario DS2 is more difficult than DS1.

Table 7: The Targets Dynamics

Scenario	DS1		DS2	
K	$a_x(k)$	$a_y(k)$	$a_x(k)$	$a_y(k)$
1-29	0	0	0	0
30-45	8	22	8	22
46-55	2	37	12	27
56-80	0	0	0	0
81-98	25	2	15	2
99-119	-2	19	-2	9
120-139	0	-1	0	-1
140-149	38	-1	28	-1
150-160	0	0	0	0

Performance measure:

The *accuracy* of the algorithms was measured in terms of position and velocity root-mean-square errors (RMSE):

$$RMS\_error(k) = \sqrt{\frac{1}{M}\sum_{j=1}^{M}(x(k) - x^j(k))(x(k) - x^j(k)^T}$$

$$\text{RMSE}_k^x = \left(\frac{1}{M}\sum_{i=1}^{M}\left(\left[x_k^i - \hat{x}_{k|k}^i\right] * \left[x_k^i - \hat{x}_{k|k}^i\right]'\right)\right)^{\frac{1}{2}}$$

$$\text{RMSE}_k^y = \left(\frac{1}{M}\sum_{i=1}^{M}\left(\left[y_k^i - \hat{y}_{k|k}^i\right] * \left[y_k^i - \hat{y}_{k|k}^i\right]'\right)\right)^{\frac{1}{2}}$$

,

$$\text{RMSE}_k^{vx} = \left(\frac{1}{M}\sum_{i=1}^{M}\left(\left[\dot{x}_k^i - \hat{\dot{x}}_{k|k}^i\right]^2 + \left[\dot{x}_k^i - \hat{\dot{x}}_{k|k}^i\right]^2\right)\right)^{\frac{1}{2}}$$

$$\text{RMSE}_k^{vy} = \left(\frac{1}{M}\sum_{i=1}^{M}\left(\left[\dot{y}_k^i - \hat{\dot{y}}_{k|k}^i\right]^2 + \left[\dot{y}_k^i - \hat{\dot{y}}_{k|k}^i\right]^2\right)\right)^{\frac{1}{2}}$$

Where

(x_k^i, y_k^i) true position, $(\dot{x}_k^i, \dot{y}_k^i)$ true velocity, $(\hat{x}_k^i, \hat{y}_k^i)$ estimated position $(\hat{\dot{x}}_k^i, \hat{\dot{y}}_k^i)$ and the estimated velocity.

The performances of the three MM tracking algorithms are investigated first over a large number of deterministic maneuver scenarios with fixed acceleration sequences. Deterministic scenarios serve to evaluate algorithms' peak errors, steady-state errors and response times. We present two of them, referred to as DS1 and DS2, in the sequel. Their acceleration values are given in Table 7. The other parameters for both scenarios are *T=1 sec, Q=0,R=1250I*

Note that while the acceleration values in DS1 are relatively close to the fixed grid points of IMM13, in DS2 they are deliberately chosen far apart from the grid points. As such, for the fixed structure estimator IMM13 the scenario DS2 is more difficult than DS1.

Table 8: The results of target dynamics in table 7

	IMM13		VIMM13		HSIMM13	
	DS1	DS2	DS1	DS2	DS1	DS2
RMSx	0.0182	0.0118	0.0.094	0.0892	5.6850	4.2039
RMSy	0.0291	0.0276	0.312	0.3131	3.6809	5.6190
RMSvx	103.3092	74.8224	105.6351	75.1330	23.8764	16.0580
RMXvy	97.7809	93.3336	103.9023	99.9368	15.2076	10.5261

Table 9: The execution Time of target dynamics in table 7

	IMM13		VIMM13		HSIMM	
	DS1	DS2	DS1	DS2	DS1	DS2
Time	0.0.0075	0.0072	0.0085	0.0073	0.0045	0.0019

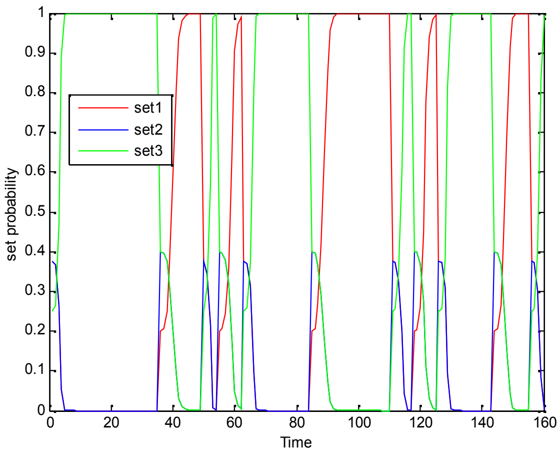

Figure 4 Activation between the sets as their set probability change for DS1 of HSIMM13

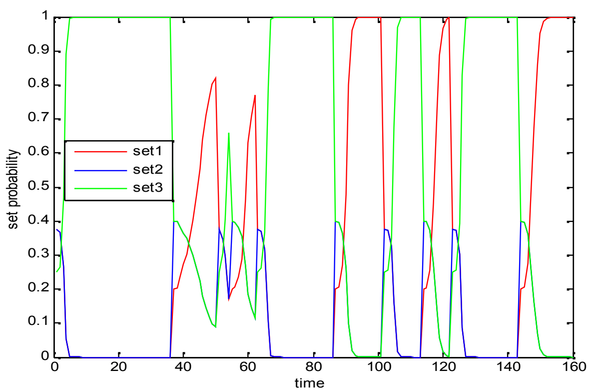

Figure 5 Activation between the sets as their set probability change for DS2 of HSIMM13

The proposed HSIMM introduced less computational time and also minimum RMS error as shown in table 5, 6,8 and 9. But it needs to be re-initialized to overcome error accumulation.

The activation between included sets is achieved by the introduced threshold value of innovation. The switching algorithm as shown in figure 4 and 5 for DS1 and DS2 is effective.

10. Conclusion

As the number of the IMM increase the algorithm stability decrease. Or in other word As the parameters change the system doesn't converge to different values. As we show in IMM with 10 models. The change of ω and ΔT . Also the choosing of their values may cause system to converge to the wrong model of ω. Also as the time step change the (increase) the system doesn't converge as ω and time step change together. Not all the models change during operating from model to another allowed.

In structure set of IMM we first choose the near values of ω in the same model to avoid converging to the wrong set. The small numbers of models increase the system stability. It doesn't diverge at the changing of time step or different values of ω. Also changing from any model to another is allowed. The advantages of the structure set of IMM are introducing varieties of motion models and also varieties of time step values. Introduce variety of Model change during operation. Introduce large number of modes of operation so we can avoid using the nonlinear models with their calibration hardness. It also introduce less computation time than introduced by the large number of IMM since we only activate the right set.

The error introduced by the structure HSIMM is due to the initialization at the beginning before converging to the right set. This error can be reduced by the refinement process if we take the saved values of the right set but it doesn't suite the real time process. The HSIMM also introduce relatively similar errors at velocity components compared to other algorithms. The computational time is minimum than introduced by IMM and VIMM. HSIMM introduces less error as the noise increase and there is no need for re adjustment to the Covariance as the noise increase so it is more robust against noise and introduces minimum computational time.

References

[1] E. Mazor, A. Averbuch, Y. Bar-Shalom, and J. Dayan. Interacting multiple model methods in target tracking: A survey. *IEEE Transactions on Aerospace and Electronic systems*, 34(1):103{123, January 1998.

[2] X.R. Li and Y. Bar-Shalom. Multiple-model estimation with variable structure. *IEEE Transactions on Automatic Control*, 41(4):478{493, April 1996.

[3] Hongshe Dang, Chongzhao Han Xi'an, Shaan and Dominique GRUYER "Combining of IMM filtering and DS data association for multi-target tracking" www.fusion2004.foi.se/papers/IF04-0876.pdf

[4] Ludmila Mihaylova and Emil Semerdjiev, "An Interacting Multiple Model Algorithm for Stochastic Systems Control" www.citeulike.org/user/sourada/article/347493 - 16k

[5] Leigh A. Johnston and Vikram Krishnamurthy, "An Improvement to the Interacting Multiple Model (IMM) Algorithm, IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 49, NO. 12, DECEMBER 2001 PP 2909-2923.

[6] Joseph J. LaViola Jr. "A Comparison of Unscented and Extended Kalman Filtering for Estimating Quaternion Motion". www.cs.brown.edu/people/jjl/pubs/laviola_acc2003.pdf -

[7] Songhwai Oh and Shankar Sastry, " An Efficient Algorithm for Tracking Multiple Maneuvering Targets". *Proc. of the IEEE International Conference on Decision and Control*, Seville, Spain, Dec. 2005.

[8] Ludmila Mihaylova 1 Emil Semerdjiev1 X. Rong Li ," Detection and Localization of Faults in System Dynamics

by IMM Estimator.
http://citeseer.ist.psu.edu/cache/papers/cs/22073/

[9] Youmin Zhang and X. Rong Li., "Detection and Diagnosis of sensor and actuator failures using IMM estimator", IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS .vol.34, No. 4 October 1998. PP 1293-1313.

[10] Donka Angelova 1 , Emil Semerdjiev 1 , Ludmila Mihaylova 1 X. Rong Li , "An IMMPDAF Solution to Benchmark Problem for Tracking in Clutter and Standoff Jammer", http://citeseer.ifi.unizh.ch/cache/papers/cs/22073/

[11] Simon J. Julier Jeffrey K. Uhlmann," A New Extension of the Kalman Filter to Nonlinear Systems". media/pdf/Julier1997_SPIE_KF.pdf/**kalman**www.cs.unc.edu/~welch/

[12] Seham Meawed Aly , Raafat El Fouly, Hoda Barak, "Extended Kalman filtering and interactive multiple model for tracking maneuvering targets in sensor networks," Intelligent solutions in embedded systems,2009 seventh workshop on, 2009 , page: 149-156

[13] X. R. Li, V. P. Jilkov, and J.-F. Ru, "Multiple-model estimation with variable structure—part VI: expected-mode augmentation," IEEE Transactions on Aerospace and Electronic Systems, 41(3): 853–867, July 2005

[14] X. R. Li, Z.-L. Zhao, and X. B. Li, "General model-set design methods for multiple-model approach," IEEE Transactions on Automatic Control, 50(9):1260–1276, September 2005

[15] X. R. Li, "Engineer's guide to variable-structure multiple-model estimation for tracking," Y. Bar-Shalom and W. D. Blair, eds., Multitarget-Multisensor Tracking: Applications and Advances, Vol. 3, Chapter 10, Artech House, Boston, 2000, pp. 499-567

[16] Xiao-Rong-Li, Yaakov Bar-Shalom, "Multiple- Model Estimation with Variable Structure " ,IEEE Transactions on Automatic Control, Vol. 41, No, 4 April 1996. pp.478-493.

[17] X. R. Li, "Engineer's guide to variable-structure multiple-model estimation for tracking," Y. Bar-Shalom and W. D. Blair, eds., Multitarget-Multisensor Tracking: Applications and Advances, Vol. 3, Chapter 10, Artech House, Boston, 2000, pp. 499-567

[18] X. R. Li, and Y. M. Zhang, "Multiple-Model Estimation With Variable Structure —Part V: Likely-Model Set Algorithm," *IEEE Trans. Aerospace and Electronic Systems*, Vol. AES-36, Apr. 2000.

[19] X. R. Li and V. P. Jilkov, "Expected-mode augmentation for multiple-model estimation," Proc. 2001 International Conf. Information Fusion, Montreal, QC, Canada, August 2001, pp. WeB1-3-WeB1-10