New Delay-dependent Stability Criteria for Linear Systems with Time-varying Delay

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Abstract
This paper is concerned with the problem of asymptotic stability for linear systems with time-varying delays. With the introduction of delay-partition approach, some new delay-dependent stability criteria are established and formulated in the form of linear matrix inequalities. Both constant time delays and time-varying delays have been taken into account. Numerical examples are given to demonstrate the effectiveness and less conservativeness of the proposed methods.

Keywords: Linear systems, Time-varying delay, Delay-partition, Asymptotic stability, Linear matrix inequalities (LMIs).

1. Introduction

Time delay is commonly encountered in various physical and engineering systems such as aircraft, biological systems, networked control systems, and so on. Since the existence of time-delays causes poor performance, oscillation, or even instability, it is very important to investigate stability analysis for systems with time-delays before designing control systems, see for example [1] and references therein.

On the other hand, neutral time-delay systems contain delays both in its state, and in its derivatives of the state. Such a system can be found in population ecology [2], distributed networks containing lossless transmission lines [3], heat exchangers, robots in contact with rigid environments [4], etc. Stability of these systems was proved to be a more complex issue because the system involves the derivative of the delayed state. Because of its wider application, the problem of the stability for neutral time-delay systems has attracted considerable attention during the last two decades.

By using the Lyapunov–Razumikhin functional approach or the Lyapunov–Krasovskii functional approach, several stability criteria have been proposed for delay-independent [5,6] and delay-dependent stability criteria [7,8] cases. Since delay independent conditions are usually more conservative than the delay-dependent conditions, more attention has been paid to the study of delay-dependent conditions. For example, a delay-dependent stability criterion for uncertain neutral systems with time-varying discrete delay was obtained in [9] based on a model transformation and Park's inequality [10]. By taking an augmented model which included the original system and the model obtained by taking the time-derivative of original system, Ariba et al. [11] proposed a new delay-dependent stability criteria for time-varying delay systems. In [12], the triple integral Lyapunov-Krasovskii functional approaches which utilize more information about states and delayed states have been proposed. Suplin et al.[13] proposed delay-dependent stability conditions for time-delay systems based on the augmented Lyapunov-Krasovskii's functional and Finsler's lemma. Therefore, it is strongly needed that some new methods should be studied to improve the upper bounds of stability criteria.

Motivated by the above, in this paper, a new delay-decomposition method for neutral systems with time-varying delays will be proposed. By constructing a suitable Lyapunov-Krasovskii's functional, some novel delay-dependent stability criteria are derived in terms of LMIs which can be solved efficiently. In order to derive less conservatively results, by using the delay decomposition approach, the delay interval [−τ,0] is decomposed into [−τ,−ατ] and [−ατ,0]. Since a tuning parameter is introduced, the information about x(t−ατ) can be taken into full consideration. Then we chosen different weighting matrices in each subinterval, which yields less
conservative delay-dependent stability criteria. Finally, numerical examples are included to show the effectiveness of the proposed method.

Notation. Throughout this paper, $R^n$ is the $n$-dimensional Euclidean space, $R^{m \times n}$ denotes the set of $m \times n$ real matrix. $X_{ij}$ denotes the element in row $i$ and column $j$ of matrix $X$. $I$ is the identity matrix. The notation * always denotes the symmetric block in one symmetric matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem statement and preliminary

Consider the following neutral system with time-varying delay:

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + A_1x(t-\tau(t)) \\
x(s) &= \phi(s), s \in [-\tau, 0]
\end{align*}
$$

(1)

where $x(t) \in R^n$ is the vector, $A, A_1$ are known constant matrices with appropriate dimensions, $\phi(s) \in C_{n,\tau}$ is a given continuous vector-valued initial function, and $\tau(t)$ is a time-varying continuous function that satisfies the conditions

$$
0 \leq \tau(t) \leq \tau \quad \tau(t) \leq \mu < 1
$$

(2)

The purpose of this paper is to establish delay-dependent stability conditions for neutral system (1). To obtain the main results, the following lemmas are needed.

Lemma 2.1[14]: For any constant matrix $X \in R^{m \times n}$, $X = X^T > 0$, a scalar function $h := h(t) > 0$, and a vector valued function $\hat{x} : [-h, 0] \rightarrow R^n$ such that the following integrations are well defined, then

$$
\begin{align*}
&\int_{-h}^{0} \hat{x}^T(t + s)X\hat{x}(t + s)ds \leq \xi_1(t) \left[ -X \begin{array}{c} X \\ X \\ -X \end{array} \right] \xi_1(t) \\
&\frac{h^2}{2} \int_{-h}^{0} \int_{-h}^{0} \hat{x}^T(s)X\hat{x}(s)dsd\theta \leq \xi_2(t) \left[ -X \begin{array}{c} X \\ X \\ -X \end{array} \right] \xi_2(t)
\end{align*}
$$

(3)

where $\xi_1(t) = [x^T(t) \quad x^T(t-h)]$ and $\xi_2(t) = [h x^T(t) \quad \int_{-h}^{t} x^T(s) ds]$

Lemma 2.2[19]: Let $f_1, f_2, \ldots, f_N : R^m \rightarrow R$ have positive values in an open subset $D$ of $R^m$. Then, the reciprocally convex combination of $f_i$ over $D$ satisfies

$$
\min_{\{a_i | a_i > 0, \sum a_i = 1\}} \sum_{i} a_i f_i(t) = \sum_{i} a_i f_i(t) + \max_{g_{ij}(t)} \sum_{i \neq j} g_{ij}(t)
$$

subject to

$$
\left\{ g_{ij} : R^m \rightarrow R, g_{ij}(t) \neq g_{ij}(t), \left[ \begin{array}{cc} f_i(t) & g_{ij}(t) \\ g_{ij}(t) & f_j(t) \end{array} \right] \geq 0 \right\}
$$

(5)

3. Main results

In this section, we propose a new delay-dependent stability criteria for neutral system (1). Both constant time delays and time-varying delays are treated. In order to obtain some less conservative sufficient conditions, we decompose the delay interval $[-\tau, 0]$ into $[-\tau, -\alpha \tau]$ and $[-\alpha \tau, 0]$, and we consider the both condition $\tau(t) \in [-\tau, -\alpha \tau]$ and $\tau(t) \in [-\alpha \tau, 0]$. For convenience, we define $e_i (i = 1, 2, \ldots, 9)$ as block entry matrices. For example, $e_1 = [0 \ 0 \ I \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$. The other notations for some vectors and matrices are defined as:

$$
\begin{align*}
&\zeta_1(t) = [x^T(t) \quad x^T(t-\tau(t)) \quad x^T(t-\alpha \tau(t)) \quad \dot{x}^T(t-\alpha \tau(t)) \quad x^T(t-\tau) \quad \int_{-\tau}^{t-\tau} x^T(s)ds] \\
&\zeta_2(t) = [x^T(t) \quad x^T(t-\tau(t)) \quad x^T(t-\tau) \quad \dot{x}^T(t-\tau) \quad x^T(t-\tau) \quad \int_{-\tau}^{t-\tau} x^T(s)ds] \\
&\zeta_0(t) = [x^T(t) \quad x^T(t-\tau(t)) \quad x^T(t-\tau) \quad \dot{x}^T(t-\tau) \quad x^T(t-\tau) \quad \int_{-\tau}^{t-\tau} x^T(s)ds] \\
&\zeta(t) = [x^T(t) \quad x^T(t-\tau) \quad x^T(t-\alpha \tau) \quad \dot{x}^T(t-\alpha \tau) \quad x^T(t-\alpha \tau) \quad \int_{-\alpha \tau}^{t-\alpha \tau} x^T(s)ds] \\
&\eta(t) = [x^T(t) \quad \dot{x}^T(t)] \\
&\Pi_0 = [e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_9] \\
&\Pi_1 = [e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_9] \\
&\Pi_2 = [e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_9] \\
&\Pi_3 = [e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_9] \\
&\Pi_4 = [e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_9]
\end{align*}
$$

Now, we have the following theorem.

Theorem 3.1: For given scalars $\tau, \mu$ and $0 < \alpha < 1$, the system (1) with (2) is asymptotically stable if there exist positive definite matrices $P = [P_{ij}]_{3 \times 3}$, $Q_1 = [Q_{ij}]_{2 \times 2}$, $Q_2 = [Q_{ij}]_{2 \times 2}$, $Q_3 = [Q_{ij}]_{2 \times 2}$, $Q_4 = [Q_{ij}]_{2 \times 2}$, $Q_5 = [Q_{ij}]_{2 \times 2}$, $Q_6 = [Q_{ij}]_{2 \times 2}$, $Q_7 = [Q_{ij}]_{2 \times 2}$, $Q_8 = [Q_{ij}]_{2 \times 2}$, $Q_9 = [Q_{ij}]_{2 \times 2}$, $Q_{10} = [Q_{ij}]_{2 \times 2}$, with appropriate dimensions such that the following LMIs hold:

$$
\begin{align*}
\Phi_k &< 0, & & \begin{bmatrix} \Psi_k & \Theta_k \\ * & \Psi_k \end{bmatrix} \geq 0, & k = 1, 2
\end{align*}
$$

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Proof: Let us consider the following candidate for the appropriate Lyapunov-Krasovskii functional:

\[
V = \sum_{i=1}^{6} V_i,
\]

where

\[
V_i = \varepsilon^T(t) P_i \varepsilon(t),
\]

\[
\begin{align*}
\Phi_1 &= \Pi_1 \Pi_1^T + \Pi_2 \Pi_2^T + (e_1 Q_{11} e_1^T + 2e_1 Q_{12} A e_1^T + A e_1 Q_{12} A^T e_1^T) \\
&\quad + (e_2 Q_{21} e_2^T + 2e_2 Q_{22} A e_2^T + A e_2 Q_{22} A^T e_2^T) \\
&\quad + (e_3 Q_{31} e_3^T + 2e_3 Q_{32} A e_3^T + A e_3 Q_{32} A^T e_3^T) \\
&\quad + (e_4 Q_{41} e_4^T + 2e_4 Q_{42} A e_4^T + A e_4 Q_{42} A^T e_4^T)
\end{align*}
\]

\[
\Phi_2 = \Pi_1 \Pi_2 + \Pi_2 \Pi_1 + (e_1 Q_{11} e_1^T + 2e_1 Q_{12} A e_1^T + A e_1 Q_{12} A^T e_1^T) \\
&\quad + (e_2 Q_{21} e_2^T + 2e_2 Q_{22} A e_2^T + A e_2 Q_{22} A^T e_2^T) \\
&\quad + (e_3 Q_{31} e_3^T + 2e_3 Q_{32} A e_3^T + A e_3 Q_{32} A^T e_3^T) \\
&\quad + (e_4 Q_{41} e_4^T + 2e_4 Q_{42} A e_4^T + A e_4 Q_{42} A^T e_4^T)
\]

\[
\Phi_3 = \Pi_1 \Pi_1^T + \Pi_2 \Pi_2^T + (e_1 Q_{11} e_1^T + 2e_1 Q_{12} A e_1^T + A e_1 Q_{12} A^T e_1^T) \\
&\quad + (e_2 Q_{21} e_2^T + 2e_2 Q_{22} A e_2^T + A e_2 Q_{22} A^T e_2^T) \\
&\quad + (e_3 Q_{31} e_3^T + 2e_3 Q_{32} A e_3^T + A e_3 Q_{32} A^T e_3^T) \\
&\quad + (e_4 Q_{41} e_4^T + 2e_4 Q_{42} A e_4^T + A e_4 Q_{42} A^T e_4^T)
\]

\[
\begin{align*}
V_1 &= \int_{-\tau}^{t-\tau} \eta^T(t) \Omega_1 \eta(t) ds + \int_{-\tau}^{t-\tau} \eta^T(t) \Omega_2 \eta(t) ds \\
V_2 &= \int_{-\tau}^{t-\tau} \eta^T(t) \Omega_1 \eta(t) ds + \int_{-\tau}^{t-\tau} \eta^T(t) \Omega_2 \eta(t) ds \\
V_3 &= \alpha \tau \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \eta^T(t) \Psi_1 \eta(t) ds d\theta \\
V_4 &= \gamma_1 \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \varepsilon^T(s) T_1 \varepsilon(s) ds d\lambda d\theta + \gamma_2 \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \varepsilon^T(s) T_2 \varepsilon(s) ds d\lambda d\theta
\end{align*}
\]

From \( V_1, V_2, V_3, \) and \( V_4, \) we have their time-derivatives as:

\[
\dot{V}_1 = 2 \varepsilon^T(t) P \varepsilon(t) \\
\dot{V}_2 = \eta^T(t) \Omega_1 \eta(t) - \eta^T(t-\tau) \Omega_1 \eta(t-\tau) \\
\dot{V}_3 = \alpha \tau \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \eta^T(t) \Psi_1 \eta(t) ds d\theta \\
\dot{V}_4 = \gamma_1 \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \varepsilon^T(s) T_1 \varepsilon(s) ds d\lambda d\theta + \gamma_2 \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \varepsilon^T(s) T_2 \varepsilon(s) ds d\lambda d\theta
\]

Also, by Eq.(4) in Lemma 2.1, we can obtain \( V_3, \) and \( V_4, \) as follows:

\[
\begin{align*}
\dot{V}_3 &= (\alpha \tau)^2 \eta^T(t) \Psi_1 \eta(t) + (1-\alpha)^2 \tau^2 \eta^T(t-\tau) \Psi_2 \eta(t-\tau) \\
&\quad - \alpha \tau \int_{-\tau}^{t-\tau} \eta^T(t) \Psi_1 \eta(t) ds - (1-\alpha)^2 \tau^2 \eta^T(t-\tau) \Psi_2 \eta(t-\tau) ds \\
\dot{V}_4 &= \gamma_1 \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \varepsilon^T(s) T_1 \varepsilon(s) ds d\theta \\
&\quad - \gamma_1 \int_{-\tau}^{t-\tau} \int_{t}^{t+\tau} \varepsilon^T(s) T_2 \varepsilon(s) ds d\theta
\end{align*}
\]
Here, we will consider the time-derivative of $V$ for two cases, $0 \leq \tau (t) \leq \alpha \tau$ and $\alpha \tau \geq \tau (t) \leq \tau$.

Case I: $0 \leq \tau (t) \leq \alpha \tau$ We can get $V_1$ as follows:

\[
\dot{V}_1 = \zeta(t) \left[ \Pi_1 \Pi_2^T + \Pi_3 \right] \zeta(t) 
\tag{15}
\]

From Eq.(11), by use of Eq.(5) in Lemma 2.2, we can get

\[
-\gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \dot{x}(s) T_1 \dot{x}(s) ds d\theta \leq \int_{-\infty}^{t} \int_{-\infty}^{\infty} \alpha \tau x(t) \left[ -T_1 \ T_1 \right] \alpha \tau x(t) ds ds \leq (1-\alpha)^2 \tau^2 e_1^T T_2 e_1^T + 2(1-\alpha) \tau e_1^T T_2 e_3^T - e_3^T T_2 e_3^T \zeta_1(t) \tag{13}
\]

(18)

Inspired by the work of [17], the following four zero equalities with any symmetric matrices $N_i (i = 1, 2, 3)$, are considered:

\[
0 = x^T(t) N_1 x(t) - x^T(t) (t - \tau(t)) N_1 x(t - \tau(t)) 
- 2 \int_{t - \tau(t)}^{t} x^T(s) N_1 x(s) ds 
0 = x^T(t - \tau(t)) N_2 x(t - \tau(t)) \tag{14}
\]

\[
- x^T(t - \tau(t)) N_2 x(t - \tau(t)) - 2 \int_{t - \tau(t)}^{t} x^T(s) N_2 x(s) ds 
0 = x^T(t - \tau(t)) N_3 x(t - \tau(t)) - x^T(t - \tau(t)) N_3 x(t - \tau(t)) \tag{16}
\]

\[
- 2 \int_{t - \tau(t)}^{t} x^T(s) N_3 x(s) ds 
\]

By use of Eq.(10) and Eq.(19), we have

\[
\dot{V}_1 = \zeta^T(t) \left[ \Pi_1 \Pi_2^T + \Pi_3 \right] \zeta(t) 
\tag{15}
\]

From Eq.(11), by use of Eq.(5) in Lemma 2.2, we can get

\[
-\alpha \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \eta^T(s) \Psi_1 \eta(s) ds = \alpha \tau \int_{t - \tau(t)}^{t} \eta^T(s) \Psi_1 \eta(s) ds 
- \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \alpha \tau \eta^T(s) \Psi_1 \eta(s) ds \leq -\alpha \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \eta^T(s) \Psi_1 \eta(s) ds \leq -\alpha \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \eta^T(s) \Psi_1 \eta(s) ds 
\tag{16}
\]

where $\Theta_1$ is the matrix satisfying $\eta^T(s) \Psi_1 \eta(s) \geq 0$. It should be noted that when $\tau (t) = 0$ or $\tau (t) = \alpha \tau$, we have $\int_{t - \tau(t)}^{t} x(s) ds = 0$ or $\int_{t - \tau(t)}^{t} x(s) ds = 0$, respectively. Thus, Eq.(15) still holds. From (11) and (16), $V_3$ satisfies:

\[
\dot{V}_3 \leq \zeta_1^T(t) \left[ (\alpha \tau)^2 e_1 W_{111} T_1 + e_1 W_{112} A_T^T + e_1 W_{122} A_T^T + e_1 W_{211} T_1 + e_1 W_{212} A_T^T + e_1 W_{222} A_T^T \right] \zeta_1(t) \tag{17}
\]

From (12), (13) and (14), $V_4$ satisfies:

\[
\dot{V}_4 \leq \zeta_1^T(t) \left[ (\alpha \tau)^2 e_1 T_1 e_1^T + 2(1-\alpha) \tau e_1 T_2 e_3^T - e_3^T T_2 e_3^T \right] \zeta_1(t) \tag{18}
\]

Here, we will consider the time-derivative of $V$ for two cases, $0 \leq \tau (t) \leq \alpha \tau$ and $\alpha \tau \geq \tau (t) \leq \tau$.

Case II: $\alpha \tau \leq \tau (t) \leq \tau$ We can get $V_1$ as follows:

\[
\dot{V}_1 = \zeta^T(t) \left[ \Pi_1 \Pi_2^T + \Pi_3 \right] \zeta(t) 
\tag{15}
\]

From Eq.(11), by use of Eq.(5) in Lemma 2.2, we can get

\[
-\alpha \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \eta^T(s) \Psi_1 \eta(s) ds = \alpha \tau \int_{t - \tau(t)}^{t} \eta^T(s) \Psi_1 \eta(s) ds 
- \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \alpha \tau \eta^T(s) \Psi_1 \eta(s) ds \leq -\alpha \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \eta^T(s) \Psi_1 \eta(s) ds \leq -\alpha \gamma_1 \int_{-\infty}^{t} \int_{-\infty}^{\infty} \eta^T(s) \Psi_1 \eta(s) ds 
\tag{16}
\]

where $\Theta_1$ is the matrix satisfying $\eta^T(s) \Psi_1 \eta(s) \geq 0$. It should be noted that when $\tau (t) = 0$ or $\tau (t) = \alpha \tau$, we have $\int_{t - \tau(t)}^{t} x(s) ds = 0$ or $\int_{t - \tau(t)}^{t} x(s) ds = 0$, respectively. Thus, Eq.(15) still holds. From (11) and (16), $V_3$ satisfies:

\[
\dot{V}_3 \leq \zeta_1^T(t) \left[ (\alpha \tau)^2 e_1 W_{111} T_1 + e_1 W_{112} A_T^T + e_1 W_{122} A_T^T + e_1 W_{211} T_1 + e_1 W_{212} A_T^T + e_1 W_{222} A_T^T \right] \zeta_1(t) \tag{17}
\]

From (12), (13) and (14), $V_4$ satisfies:

\[
\dot{V}_4 \leq \zeta_1^T(t) \left[ (\alpha \tau)^2 e_1 T_1 e_1^T + 2(1-\alpha) \tau e_1 T_2 e_3^T - e_3^T T_2 e_3^T \right] \zeta_1(t) \tag{18}
\]
where \( \Theta_2 \) is the matrix satisfying \[ \begin{bmatrix} \Psi_2 & \Theta_2 \\ * & \Psi_2 \end{bmatrix} \geq 0. \] It should be noted that when \( \tau(t) = \alpha \tau \) or \( \tau(t) = \tau \), we have \[ \int_{t-\tau}^{t-\alpha \tau} x(s)ds = 0 \] or \[ \int_{t-\tau}^{t-\tau(t)} x(s)ds = 0 \], respectively. Thus, Eq. (22) still holds. From (11) and (22), \( V_3 \) satisfies:

\[ V_3 \leq \zeta(t)^T \begin{bmatrix} \alpha_1 e_{W_{1,11}}^T e_1^T + 2e_1 W_{1,12} A_T^T + A e_{W_{1,22}} A_T^T \\
+ (1-\alpha)^2 \tau^2 e_1 W_{2,11} e_1^T + 2e_1 W_{2,12} A_T^T + A e_{W_{2,22}} A_T^T \\
- \epsilon_7 W_{1,11} e_1^T - 2e_1 W_{1,12} e_1^T + 2e_1 W_{1,22} e_1^T \\
+ 2e_1 W_{1,22} e_1^T - e_3 W_{1,22} e_1^T + \Pi_2^T \begin{bmatrix} \Psi_2 & \Theta_2 \\ * & \Psi_2 \end{bmatrix} \Pi_3^T \zeta_1 (t) \end{bmatrix} \]

(23)

From (12), (13) and (14), \( V_4 \) satisfies:

\[ V_4 \leq \zeta(t)^T A e_{S_1} \zeta(t) + \alpha \epsilon_7 e_1^T - e_3 W_{1,22} e_1^T + \epsilon_7 W_{1,11} e_1^T - 2e_1 W_{1,12} e_1^T + 2e_1 W_{1,22} e_1^T \]

(24)

Inspired by the work of [17], the following four zero equalities with any symmetric matrices \( N_j, (i = 4, 5, 6) \), are considered:

\[ 0 = x^T(t) N_4 x(t) - x^T(t - \alpha \tau) N_4 x(t - \alpha \tau) - 2 \int_{t-\alpha \tau}^{t} x^T(s) N_4 \dot{x}(s) ds \]

\[ 0 = x^T(t - \tau(t)) N_5 x(t - \tau(t)) - x^T(t - \tau(t)) N_5 x(t - \tau(t)) - 2 \int_{t-\tau(t)}^{t} x^T(s) N_5 \dot{x}(s) ds \]

\[ 0 = x^T(t - \tau(t)) N_6 x(t - \tau(t)) - x^T(t - \tau(t)) N_6 x(t - \tau(t)) - 2 \int_{t-\tau(t)}^{t} x^T(s) N_6 (t) \dot{x}(s) ds \]

(25)

By use of Eq. (10) and Eq. (25), we have

\[ \dot{V}_6 \leq \zeta(t)^T \begin{bmatrix} \alpha_1 e_{Q_1} + \epsilon_1 Q_1 e_1^T + \epsilon_2 e_2 W_{1,11} e_1^T + 2e_1 W_{1,12} e_1^T + A e_{W_{1,22}} A_T^T \\
+ e_1 W_{1,11} e_1^T - e_2 W_{1,12} e_1^T + e_2 W_{1,22} e_1^T + e_3 W_{2,22} e_1^T - e_4 W_{1,22} e_1^T \\
- \epsilon_7 W_{1,11} e_1^T - 2e_1 W_{1,12} e_1^T + 2e_1 W_{1,22} e_1^T \\
+ 2e_1 W_{1,22} e_1^T - e_3 W_{1,22} e_1^T + \Pi_1^T \begin{bmatrix} \Psi_1 & \Theta_1 \\ * & \Psi_1 \end{bmatrix} \Pi_3^T \zeta_1 (t) \end{bmatrix} \]

Then combining Eqs. (8)-(9), (21), (23)-(24), (26) yields \( \dot{V} \leq \zeta(t)^T \Phi_1^{(2)} \zeta(t) \). If \( \Phi_1^{(2)} < 0 \) and \( \alpha \tau \leq \tau \), then \( \dot{V} < 0 \), the system (1) is asymptotically stable. Thus, the proof is completed.

Remark 3.2: In order to reduce the conservatism, a new delay-dependent stability criterion is obtained in Theorem 3.1 by constructing a new Lyapunov-Krasovskii functional. In Eq. (6), \( V_2, V_3 \) and \( V_4 \) are constructed by using such an idea that the whole delay interval \([0, \tau]\) is decomposed into two partitions. We consider the time-varying delay \( \tau(t) \) in each partition, then on each partition we choose different weighting matrices, which yields less conservative delay-dependent stability criteria, and that will be illustrated through the examples in the next section.

Remark 3.3: Recently, the reciprocally convex optimization technique was proposed in [15] and [17] to reduce the conservatism of stability criteria for systems with time-varying delays. Motivated by this work, the proposed methods of [15] and [17] were applied to the delay-decomposition method as shown in Eq. (15) and (20). In many cases, the information on the delay derivative may not be available. Considering this case, the following result can be obtained from Theorem 3.1 by omitting \( V_5 \).

Corollary 3.4: For given scalars \( \tau \) and \( 0 < \alpha < 1 \), the system (1) with (2) is asymptotically stable if there exist positive definite matrices \( P_1, P_2, P_3, P_4, P_5, P_6 \), with appropriate dimensions such that the following LMIs hold:

\[ \begin{bmatrix} \Psi_k & \Theta_k \\ * & \Psi_k \end{bmatrix} \geq 0, k = 1, 2, \]

\[ \begin{bmatrix} Q_3 N_1 & * \\ Q_4 & * \end{bmatrix} > 0, \begin{bmatrix} Q_5 N_3 & * \\ Q_6 & * \end{bmatrix} > 0, \begin{bmatrix} Q_7 N_5 & * \\ Q_8 & * \end{bmatrix} > 0, \]

where \( \Phi_1^{(2)} = \Pi_1^T \begin{bmatrix} P_1 & P_2 \\ P_2^T & \Pi_2^T \end{bmatrix} + (e_1 W_{1,11} e_1^T + 2e_1 W_{1,12} e_1^T + A e_{W_{1,22}} A_T^T \\
+ e_2 W_{1,11} e_1^T - e_2 W_{1,12} e_1^T + e_2 W_{1,22} e_1^T + e_3 W_{2,22} e_1^T - e_4 W_{1,22} e_1^T \\
- \epsilon_7 W_{1,11} e_1^T - 2e_1 W_{1,12} e_1^T + 2e_1 W_{1,22} e_1^T \\
+ 2e_1 W_{1,22} e_1^T - e_3 W_{1,22} e_1^T + \Pi_1^T \begin{bmatrix} \Psi_1 & \Theta_1 \\ * & \Psi_1 \end{bmatrix} \Pi_3^T \zeta_1 (t) \end{bmatrix} \]

(26)

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Proof. The proof of this corollary immediately follows from Theorem 3.1.

When \( \tau (t) \) is constant: \( \tau (t) = \tau \), we have the following theorem.

**Theorem 3.5**: For given scalars \( \tau < \alpha < 1 \), the system (1) with (2) is asymptotically stable if there exist positive definite matrices \( P = [P_1 \ldots P_6] \), \( \Omega = [\Omega_1 \ldots \Omega_6] \), \( \Psi_1 = [W_1 \ldots W_4] \), \( \Psi_2 = [W_5 \ldots W_8] \), \( T_j, Q_i \) (i = 3, 4, 5, 6), any matrices \( N_i \) (i = 1, 2), with appropriate dimensions such that the following LMIs hold:

\[
+ \alpha \tau (e_1 Q_3 e_1^T + A_1 e_2 Q_4 A_4 e_0^T) + e_1 N_1 e_1^T - e_2 N_2 e_2^T
- e_3 N_3 e_3^T + (1 - \alpha) \tau (e_1 Q_5 e_1^T + A_1 e_2 Q_6 A_4 e_0^T)
- e_4 T_1 e_1^T + A_0 (e_1 Q_1 e_1^T + e_2 Q_2 e_2^T)
- e_5 T_2 e_2^T - (1 - \alpha) \tau (e_1 T_1 e_1^T + e_2 T_2 e_2^T)
- (1 - \alpha)^2 \tau^2 e_1 e_1^T + 2 (1 - \alpha) \tau e_1 T_1 e_1^T
\]

Proof. Let us consider the following candidate for the appropriate Lyapunov-Krasovskii functional:

\[
V = \sum_{i=1}^{6} V_i,
\]

where

\[
V_i = \eta^T_i(t) \Omega_i \eta_i(t) ds + \int_{t}^{t-\alpha t} \eta^T_i(t) \Omega_i \eta_i(t) ds d\theta,
\]

\[
V_3 = \alpha \tau \int_{t-\alpha t}^{t} \eta^T_i(t) \Psi_i \eta_i(t) ds d\theta,
\]

\[
V_4 = \gamma_1 \int_{t-\alpha t}^{t} \int_{t-\alpha t}^{t} \dot{x}^T(s) T_1 \dot{x}(s) ds d\theta d\lambda,
\]

\[
V_5 = \gamma_1 \int_{t-\alpha t}^{t} \int_{t-\alpha t}^{t} \dot{x}^T(s) Q_3 \dot{x}(s) + \dot{x}^T(s) Q_4 \dot{x}(s) ds d\theta d\lambda.
\]

From \( V_i, V_j \), we have their time-derivatives as:

\[
\dot{V}_1 = 2 \alpha \tau (e_1 e_1^T + A_0 e_2 A_4 e_0^T)
+ (1 - \alpha)^2 \tau (e_1 Q_1 e_1^T + e_2 Q_2 e_2^T)
+ e_1 T_1 e_1^T + A_0 (e_1 Q_1 e_1^T + e_2 Q_2 e_2^T)
+ e_5 T_2 e_2^T - (1 - \alpha) \tau (e_1 T_1 e_1^T + e_2 T_2 e_2^T)
+ (1 - \alpha)^2 \tau^2 e_1 e_1^T + 2 (1 - \alpha) \tau e_1 T_1 e_1^T
\]

By Eq. (4) in Lemma 2.1, we can obtain \( \dot{V}_3 \) as follows:

\[
\dot{V}_3 = (1 - \alpha)^2 \tau^2 e_1 e_1^T + 2 (1 - \alpha) \tau e_1 T_1 e_1^T
+ (1 - \alpha)^2 \tau (e_1 Q_1 e_1^T + e_2 Q_2 e_2^T)
+ e_1 T_1 e_1^T + A_0 (e_1 Q_1 e_1^T + e_2 Q_2 e_2^T)
+ e_5 T_2 e_2^T - (1 - \alpha) \tau (e_1 T_1 e_1^T + e_2 T_2 e_2^T)
+ (1 - \alpha)^2 \tau^2 e_1 e_1^T + 2 (1 - \alpha) \tau e_1 T_1 e_1^T
\]
Also, we can get $V_1$ as follows:

$$V_1 = \gamma_1^2 \int_0^t \dot{x}(t) T_1 \dot{x}(t) + \gamma_2^2 \int_{t-\tau}^t \dot{x}(t) T_2 \dot{x}(t) ds$$

and

$$-\gamma_1 \int_0^t \int_{t-\tau}^t \dot{x}(t) T_1 \dot{x}(s) ds \, d\theta$$

$$-\gamma_2 \int_{t-\tau}^t \int_{t-\tau-\theta}^t \dot{x}(t) T_2 \dot{x}(s) ds \, d\theta$$

(30)

Then combining Eqs.(27)-(29), (31) and (34) yields

$$\dot{V}_1 \leq \xi_0^T(t) [\alpha \tau (e_1 Q_3 e_1^T + A_{00} Q_4 A_{00}^T) + (1-\alpha) \tau (e_1 Q_5 e_1^T + A_{00} Q_6 A_{00}^T) + e_1 N_1 e_1^T - e_2 N_2 e_2^T - e_3 N_3 e_3^T] \xi_0(t)$$

(34)

4. Numerical examples

In this section, we provide two examples to show the less conservativeness of the proposed new stability criteria in this paper.

**Example 1.** Consider the following neutral time-delay system

$$\dot{x}(t) = A(t) x(t) + A_1 x(t-\tau(t))$$

with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

If $\mu < 1$, applying Theorem 3.1, the corresponding maximum admissible upper bounds are given in Table 1 which clearly shows that the effectiveness of the delay-decomposition approach.

**Example 2.** Consider the following nominal neutral system with constant time-delay

$$\dot{x}(t) = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix} \int_0^t x(t-\tau) d\tau$$

From $V_1$ we can obtain

$$\dot{V}_1 \leq \xi_0^T(t) [\alpha \tau (e_1 Q_3 e_1^T + A_{00} Q_4 A_{00}^T) + (1-\alpha) \tau (e_1 Q_5 e_1^T + A_{00} Q_6 A_{00}^T) + e_1 N_1 e_1^T - e_2 N_2 e_2^T - e_3 N_3 e_3^T] \xi_0(t)$$

Inspired by the work of [17], the following four zero equalities with any symmetric matrices $N_i (i=1,2)$, are considered:

$$0 = x(t) N_1 x(t) - x(t-\tau) N_1 x(t-\tau)$$

$$-2 \int_{t-\tau}^t x(t) N_1 \dot{x}(s) ds$$

$$0 = x(t) N_2 x(t-\tau) - x(t-\tau) N_2 x(t-\tau)$$

$$-2 \int_{t-\tau}^t x(t) N_2 \dot{x}(s) ds$$

(33)

By use of Eq.(32) and Eq.(33), we have

$$\dot{V}_1 \leq \xi_0^T(t) [\alpha \tau (e_1 Q_3 e_1^T + A_{00} Q_4 A_{00}^T) + (1-\alpha) \tau (e_1 Q_5 e_1^T + A_{00} Q_6 A_{00}^T) + e_1 N_1 e_1^T - e_2 N_2 e_2^T - e_3 N_3 e_3^T] \xi_0(t)$$

5. Conclusion

In this paper, new delay-dependent stability criteria for neutral time-delay systems are proposed. In order to obtain less conservative results, a new delay-decomposition method is used to improve the maximum admissible upper bounds of stability criterion. Numerical examples have been considering.
given to show that our stability are less conservative than some existing ones in the literatures.

**References**


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Table 1: The maximum admissible upper bounds of time-varying delays with different values of $\mu$ (Example1)

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>0</th>
<th>0.1</th>
<th>0.5</th>
<th>0.9</th>
<th>unknown</th>
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<tr>
<td>[18]</td>
<td>4.47</td>
<td>3.60</td>
<td>2.00</td>
<td>1.18</td>
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</tr>
<tr>
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<td>3.60</td>
<td>2.00</td>
<td>1.18</td>
<td>-</td>
</tr>
<tr>
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<td>1.18</td>
<td>-</td>
</tr>
<tr>
<td>[20]</td>
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<td>2.04</td>
<td>1.37</td>
<td>-</td>
</tr>
<tr>
<td>[21]</td>
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<td>1.87</td>
<td>-</td>
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<td>2.40</td>
<td>2.12</td>
<td>2.12</td>
</tr>
<tr>
<td>Theorem 3.1</td>
<td>5.87 $(\alpha = 0.59)$</td>
<td>4.43 $(\alpha = 0.46)$</td>
<td>2.46 $(\alpha = 0.22)$</td>
<td>2.22 $(\alpha = 0.51)$</td>
<td>2.22 $(\alpha = 0.51)$</td>
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Table 2: The maximum admissible upper bounds of constant time delays (Example2)

<table>
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<th>Method</th>
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<td>[25]</td>
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<td>[26]</td>
<td>1.9132</td>
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<tr>
<td>[27]</td>
<td>2.0054</td>
</tr>
<tr>
<td>[16]</td>
<td>2.1046</td>
</tr>
<tr>
<td>Theorem 3.5</td>
<td>2.1445 $\alpha = 0.57$</td>
</tr>
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