New Delay-dependent Stability Criteria for Linear Systems with Time-varying Delay

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Abstract

This paper is concerned with the problem of asymptotic stability for linear systems with time-varying delays. With the introduction of delay-partition approach, some new delay-dependent stability criteria are established and formulated in the form of linear matrix inequalities. Both constant time delays and time-varying delays have been taken into account. Numerical examples are given to demonstrate the effectiveness and less conservativeness of the proposed methods.

Keywords: *Linear systems, Time-varying delay, Delay-partition, Asymptotic stability, Linear matrix inequalities(LMIs).*

1. Introduction

Time delay is commonly encountered in various physical and engineering systems such as aircraft, biological systems, networked control systems, and so on. Since the existence of time-delays causes poor performance, oscillation, or even instability, it is very important to investigate stability analysis for systems with time-delays before designing control systems, see for example [1] and references therein.

On the other hand, neutral time-delay systems contain delays both in its state, and in its derivatives of the state. Such a system can be found in population ecology [2], distributed networks containing lossless transmission lines [3], heat exchangers, robots in contact with rigid environments [4], etc. Stability of these systems was proved to be a more complex issue because the system involves the derivative of the delayed state. Because of its wider application, the problem of the stability for neutral time-delay systems has attracted considerable attention during the last two decades.

By using the Lyapunov--Razumikhin functional approach or the Lyapunov--Krasovskii functional approach, several stability criteria have been proposed for delay-independent [5,6] and delay-dependent stability criteria [7,8] cases. Since delay independent conditions are usually more conservative than the delay-dependent conditions, more attention has been paid to the study of delay-dependent conditions. For example, a delay-dependent stability criterion for uncertain neutral systems with time-varying discrete delay was obtained in [9] based on a model transformation and Park's inequality [10].By taking an augmented model which included the original system and the model obtained by taking the time-derivative of original system, Ariba et al. [11] proposed a new delay-dependent stability criteria for time-varying delay systems. In [12], the triple integral Lyapunov-Krasovskii functional approaches which utilize more information about states and delayed states have been proposed. Suplin et al.[13] proposed delay-dependent stability conditions for time-delay systems based on the augmented Lyapunov-Krasovskii's functional and Finsler's lemma. Therefore, it is strongly needed that some new methods should be studied to improve the upper bounds of stability criteria.

Motivated by the above, in this paper, a new delay-decomposition method for neutral systems with time-varying delays will be proposed. By constructing a suitable Lyapunov-Krasovskii's functional, some novel delay-dependent stability criteria are derived in terms of LMIs which can be solved efficiently. In order to derive less conservatively results, by using the delay decomposition approach, the delay interval $[-\tau,0]$ is decomposed into $[-\tau,-\alpha\tau]$ and $[-\alpha\tau,0]$. Since a tuning parameter is introduced, the information about $x(t-\alpha\tau)$ can be taken into full consideration. Then we chosen different weighting matrices in each subinterval, which yields less

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conservative delay-dependent stability criteria. Finally, numerical examples are included to show the effectiveness of the proposed method.

Notation. Throughout this paper, R^n is the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of $m\times n$ real matrix. X_{ij} denotes the element in row i and column j of matrix X . I is the identity matrix. The notation $*$ always denotes the symmetric block in one symmetric matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem statement and preliminary

Consider the following neutral system with time-varying delay:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1 x(t-\tau(t)) \\ x(s) &= \phi(s), s\in[-\tau,0] \end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the vector, A, A_1 are known constant matrices with appropriate dimensions, $\phi(s)\in C_{n,\tau}$ is a given continuous vector-valued initial function, and $\tau(t)$ is a time-varying continuous function that satisfies the conditions

$$0\le\tau(t)\le\tau \quad \dot{\tau}(t)\le\mu<1 \tag{2}$$

The purpose of this paper is to establish delay-dependent stability conditions for neutral system (1). To obtain the main results, the following lemmas are needed.

Lemma 2.1[14]: For any constant matrix $X\in R^{n\times n}$, $X = X^T > 0$, a scalar function $h := h(t) > 0$, and a vector valued function $\dot{x}:[-h,0]\to R^n$ such that the following integrations are well defined, then

$$-h\int_{-h}^{0}\dot{x}^T(t+s)X\dot{x}(t+s)ds \le \xi_1^T(t)\begin{bmatrix} -X & X \\ X & -X \end{bmatrix}\xi_1(t) \tag{3}$$

$$-\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)X\dot{x}(s)dsd\theta \le \xi_2^T(t)\begin{bmatrix} -X & X \\ X & -X \end{bmatrix}\xi_2(t) \tag{4}$$

where $\xi_1^T(t) = [x^T(t) \quad x^T(t-h)]$

and $\xi_2^T(t) = [h\,x^T(t) \quad \int_{t-h}^{t} x^T(s)\,ds]$

Lemma 2.2[19]: Let $f_1, f_2,..., f_N : R^m \mapsto R$ have positive values in an open subset D of R^m . Then, the reciprocally convex combination of f_i over D satisfies

$$\min_{\{\alpha_i|\alpha_i>0,\sum_i\alpha_i=1\}}\sum_i\frac{1}{\alpha_i}f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)}\sum_{i\neq j}g_{i,j}(t)$$

subject to

$$\left\{g_{i,j}:R^m\longmapsto R, g_{j,i}(t)\triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix}\ge 0\right\} \tag{5}$$

3. Main results

In this section, we propose a new delay-dependent stability criteria for neutral system (1). Both constant time delays and time-varying delays are treated. In order to obtain some less conservative sufficient conditions, we decompose the delay interval $[-\tau,0]$ into $[-\tau,-\alpha\tau]$ and $[-\alpha\tau,0]$, and we consider the both condition $\tau(t)\in[-\tau,-\alpha\tau]$ and $\tau(t)\in[-\alpha\tau,0]$. For convenience, we define $e_i (i=1,2,...,9)$ as block entry matrices. For example, $e_3^T = [0 \quad 0 \quad I \quad 0 \quad ... \quad 0]$. The other notations for some vectors and matrices are defined as:

$\zeta_1^T(t)=[x^T(t)\, x^T(t-\tau(t))\, x^T(t-\alpha\tau)\, x^T(t-\tau)\, \dot{x}^T(t-\alpha\tau)$
$\dot{x}^T(t-\tau)\int_{t-\tau(t)}^{t} x^T(s)ds \int_{t-\alpha\tau}^{t-\tau(t)} x^T(s)ds \int_{t-\tau}^{t-\alpha\tau} x^T(s)ds]$,

$\zeta_2^T(t)=[x^T(t)\, x^T(t-\tau(t))\, x^T(t-\alpha\tau)\, x^T(t-\tau)\, \dot{x}^T(t-\alpha\tau)$
$\dot{x}^T(t-\tau)\int_{t-\alpha\tau}^{t} x^T(s)ds\int_{t-\tau(t)}^{t-\alpha\tau} x^T(s)ds \int_{t-\tau}^{t-\tau(t)} x^T(s)ds]$,

$\zeta_0^T(t)=[x^T(t)\; x^T(t-\alpha\tau)\, x^T(t-\tau)\, \dot{x}^T(t-\alpha\tau)\, \dot{x}^T(t-\tau)$
$\int_{t-\alpha\tau}^{t} x^T(s)ds \int_{t-\tau}^{t-\alpha\tau} x^T(s)ds]$,

$\varepsilon^T(t)=[x^T(t)\, x^T(t-\alpha\tau)\, x^T(t-\tau)\int_{t-\alpha\tau}^{t} x^T(s)ds$
$\int_{t-\tau}^{t-\alpha\tau} x^T(s)ds]$

$\eta^T(t)=[x^T(t)\,\dot{x}^T(t)]$

$\Pi_0^1=[e_1\, e_2\, e_3\, e_6\, e_7]$, $\Pi_0^2=[A_{c0}\, e_4\, e_5\, e_1-e_3\, e_3-e_4]$,

$\Pi_1^2=[e_1\, e_3\, e_4\, e_7\, e_8+e_9]$, $\Pi_1^1=[e_1\, e_3\, e_4\, e_7+e_8\, e_9]$

$\Pi_3^2=[e_8\;\, e_3-e_2\;\, e_9\;\, e_2-e_4]$,

$\Pi_2=[A_c\, e_6\, e_7\, e_1-e_3\, e_3-e_4]$, $\Pi_3^1=[e_7\;\, e_1-e_2\;\, e_8\;\, e_2-e_3]$.

$A_c=[A\, A_1\, 0\,0\,0\,0\,0\,0\,0]$, $A_{c0}=[A\, A_1\, 0\,0\,0\,0\,0]$

Now, we have the following theorem.

Theorem 3.1: For given scalars τ , μ and $0<\alpha<1$, the system (1) with (2) is asymptotically stable if there exist positive definite matrices $P=[P_{ij}]_{5\times5}$, $\Omega_1=[Q_{1,ij}]_{2\times2}$, $\Omega_2=[Q_{2,ij}]_{2\times2}$, $\Psi_1=[W_{1,ij}]_{2\times2}$, $\Psi_2=[W_{2,ij}]_{2\times2}$, T_j, Q_0, $Q_i(i=3,4,5,6)$, any matrices $\Theta_1=[S_{1,ij}]_{2\times2}$, $\Theta_2=[S_{2,ij}]_{2\times2}$, $N_i(i=1,2,3,4,5,6)$, with appropriate dimensions such that the following LMIs hold:

$$\Phi_1^{(k)}<0, \begin{bmatrix} \Psi_k & \Theta_k \\ * & \Psi_k \end{bmatrix}\ge 0, k=1,2$$

$$\begin{bmatrix} Q_3 & N_1 \\ * & Q_4 \end{bmatrix} > 0, \begin{bmatrix} Q_3 & N_2 \\ * & Q_4 \end{bmatrix} > 0, \begin{bmatrix} Q_5 & N_3 \\ * & Q_6 \end{bmatrix} > 0,$$
$$\begin{bmatrix} Q_3 & N_4 \\ * & Q_4 \end{bmatrix} > 0, \begin{bmatrix} Q_5 & N_5 \\ * & Q_6 \end{bmatrix} > 0, \begin{bmatrix} Q_5 & N_6 \\ * & Q_6 \end{bmatrix} > 0,$$

where

$$\begin{aligned}
\Phi_1^{(1)}=&\Pi_1^1 P\Pi_2^T+\Pi_2 P(\Pi_1^1)^T+(e_1Q_{1,11}e_1^T+2e_1Q_{1,12}A_c^T+A_cQ_{1,22}A_c^T)\\
&+(e_3Q_{2,11}e_3^T+2e_3Q_{2,12}e_5^T+e_5Q_{2,22}e_5^T)\\
&-(e_4Q_{2,11}e_4^T+2e_4Q_{2,12}e_6^T+e_6Q_{2,22}e_6^T)\\
&+(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_c^T+A_cW_{1,22}A_c^T)\\
&+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_c^T+A_cW_{2,22}A_c^T)\\
&-e_9W_{2,11}e_9^T-2e_9W_{2,12}e_3^T+2e_9W_{2,12}e_4^T-e_3W_{2,22}e_3^T\\
&+2e_3W_{2,22}e_4^T-e_4W_{2,22}e_4^T-\Pi_3^1\begin{bmatrix} \Psi_1 & \Theta_1 \\ * & \Psi_1 \end{bmatrix}(\Pi_3^1)^T\\
&+A_c(\gamma_1^2T_1+\gamma_2^2T_2)A_c^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_7^T\\
&-e_7T_1e_7^T+e_1Q_0e_1^T-(1-u)e_2Q_0e_2^T-e_7T_1e_7^T\\
&-2e_7T_1e_8^T-e_8T_1e_8^T-(1-\alpha)^2\tau^2e_1T_2e_1^T-e_9T_2e_9^T\\
&+2(1-\alpha)\tau e_1T_2e_9^T+\alpha\tau(e_1Q_3e_1^T+A_cQ_4A_c^T)\\
&-(e_3Q_{1,11}e_3^T+2e_3Q_{1,12}e_5^T+e_5Q_{1,22}e_5^T)\\
&+(1-\alpha)\tau(e_1Q_5e_1^T+A_cQ_6A_c^T)+e_1N_1e_1^T\\
&-e_2N_1e_2^T+e_2N_2e_2^T-e_3N_2e_3^T+e_3N_3e_3^T-e_4N_3e_4^T
\end{aligned}$$

$$\begin{aligned}
\Phi_1^{(2)}=&\Pi_1^2 P\Pi_2^T+\Pi_2 P(\Pi_1^2)^T+(e_1Q_{1,11}e_1^T+2e_1Q_{1,12}A_c^T+A_cQ_{1,22}A_c^T)\\
&+(e_3Q_{2,11}e_3^T+2e_3Q_{2,12}e_5^T+e_5Q_{2,22}e_5^T)\\
&-(e_4Q_{2,11}e_4^T+2e_4Q_{2,12}e_6^T+e_6Q_{2,22}e_6^T)\\
&+(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_c^T+A_cW_{1,22}A_c^T)\\
&+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_c^T+A_cW_{2,22}A_c^T)\\
&-e_7W_{1,11}e_7^T-2e_7W_{1,12}e_1^T+2e_7W_{1,12}e_3^T-e_1W_{1,22}e_1^T\\
&+2e_1W_{1,22}e_3^T-e_3W_{1,22}e_3^T-\Pi_3^2\begin{bmatrix} \Psi_2 & \Theta_2 \\ * & \Psi_2 \end{bmatrix}(\Pi_3^2)^T\\
&+A_c(\gamma_1^2T_1+\gamma_2^2T_2)A_c^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_7^T\\
&-e_7T_1e_7^T+e_1Q_0e_1^T-(1-u)e_2Q_0e_2^T\\
&-(1-\alpha)^2\tau^2e_1T_2e_1^T+2(1-\alpha)\tau(e_1T_2e_8^T+e_1T_2e_9^T)\\
&-e_8T_2e_8^T-2e_8T_2e_9^T-e_9T_2e_9^T+e_1N_4e_1^T-e_2N_5e_2^T\\
&+\alpha\tau(e_1Q_3e_1^T+A_cQ_4A_c^T)+(1-\alpha)\tau(e_1Q_5e_1^T+A_cQ_6A_c^T)\\
&-(e_3Q_{1,11}e_3^T+2e_3Q_{1,12}e_5^T+e_5Q_{1,22}e_5^T)\\
&+e_2N_6e_2^T-e_3N_4e_3^T+e_3N_5e_3^T-e_4N_6e_4^T
\end{aligned}$$

Proof: Let us consider the following candidate for the appropriate Lyapunov-Krasovskii functional:

$$V=\sum_{i=1}^{6}V_i, \tag{6}$$

where

$$\begin{aligned}
V_1=&\varepsilon^T(t)P\varepsilon(t),\\
V_2=&\int_{t-\alpha\tau}^{t}\eta^T(t)\Omega_1\eta(t)ds+\int_{t-\tau}^{t-\alpha\tau}\eta^T(t)\Omega_2\eta(t)ds,\\
V_3=&\alpha\tau\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}\eta^T(t)\Psi_1\eta(t)dsd\theta\\
&+(1-\alpha)\tau\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}\eta^T(t)\Psi_2\eta(t)dsd\theta,\\
V_4=&\gamma_1\int_{-\alpha\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsd\lambda d\theta\\
&+\gamma_2\int_{-\tau}^{-\alpha\tau}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsd\lambda d\theta,\\
V_5=&\int_{t-\tau(t)}^{t}x^T(s)Q_0x(s)ds,\\
V_6=&\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}[x^T(s)Q_3x(s)+\dot{x}^T(s)Q_4\dot{x}(s)]dsd\theta\\
&+\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}[x^T(s)Q_5x(s)+\dot{x}^T(s)Q_6\dot{x}(s)]dsd\theta,
\end{aligned}$$

$$\gamma_1=\frac{(\alpha\tau)^2}{2},\ \gamma_2=\frac{(1-\alpha)^2\tau^2}{2}$$

From V_1, V_2, V_5, and V_6, we have their time-derivatives as:

$$\dot{V}_1=2\varepsilon^T(t)P\dot{\varepsilon}(t) \tag{7}$$

$$\begin{aligned}
\dot{V}_2=&\eta^T(t)\Omega_1\eta(t)-\eta^T(t-\alpha\tau)\Omega_1\eta(t-\alpha\tau)\\
&+\eta^T(t-\alpha\tau)\Omega_2\eta(t-\alpha\tau)-\eta^T(t-\tau)\Omega_2\eta(t-\tau)\\
=&\zeta_1^T(t)[(e_1Q_{1,11}e_1^T+2e_1Q_{1,12}A_c^T+A_cQ_{1,22}A_c^T)\\
&-(e_3Q_{1,11}e_3^T+2e_3Q_{1,12}e_5^T+e_5Q_{1,22}e_5^T)\\
&+(e_3Q_{2,11}e_3^T+2e_3Q_{2,12}e_5^T+e_5Q_{2,22}e_5^T)\\
&-(e_4Q_{2,11}e_4^T+2e_4Q_{2,12}e_6^T+e_6Q_{2,22}e_6^T)]\zeta_1(t)
\end{aligned} \tag{8}$$

$$\dot{V}_5\leq\zeta_1^T(t)[e_1Q_0e_1^T-(1-u)e_2Q_0e_2^T]\zeta_1(t) \tag{9}$$

$$\begin{aligned}
\dot{V}_6\leq&\zeta_1^T(t)[\alpha\tau(e_1Q_3e_1^T+A_cQ_4A_c^T)\\
&+(1-\alpha)\tau(e_1Q_5e_1^T+A_cQ_6A_c^T)]\zeta_1(t)\\
&-\int_{t-\alpha\tau}^{t}[x^T(s)Q_3x(s)+\dot{x}^T(s)Q_4\dot{x}(s)]ds\\
&-\int_{t-\tau}^{t-\alpha\tau}[x^T(s)Q_5x(s)+\dot{x}^T(s)Q_6\dot{x}(s)]ds
\end{aligned} \tag{10}$$

Also, by Eq.(4) in Lemma 2.1, we can obtain $\dot{V}_3$, and $\dot{V}_4$ as follows:

$$\begin{aligned}
\dot{V}_3=&(\alpha\tau)^2\eta^T(t)\Psi_1\eta(t)+(1-\alpha)^2\tau^2\eta^T(t)\Psi_2\eta(t)\\
&-\alpha\tau\int_{t-\alpha\tau}^{t}\eta^T(s)\Psi_1\eta(s)ds-(1-\alpha)\tau\int_{t-\tau}^{t-\alpha\tau}\eta^T(s)\Psi_2\eta(s)ds
\end{aligned} \tag{11}$$

$$\begin{aligned}
\dot{V}_4=&\gamma_1^2\dot{x}^T(t)T_1\dot{x}(t)+\gamma_2^2\dot{x}^T(t)T_2\dot{x}(t)\\
&-\gamma_1\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsd\theta\\
&-\gamma_2\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsd\theta
\end{aligned} \tag{12}$$

and

$$-\gamma_1\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsd\theta\leq \begin{bmatrix}\alpha\tau x(t)\\ \int_{t-\alpha\tau}^{t}x(s)ds\end{bmatrix}^T\begin{bmatrix}-T_1 & T_1\\ * & -T_1\end{bmatrix}\begin{bmatrix}\alpha\tau x(t)\\ \int_{t-\alpha\tau}^{t}x(s)ds\end{bmatrix} \tag{13}$$

$$-\gamma_2\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsd\theta\leq \begin{bmatrix}(1-\alpha)\tau x(t)\\ \int_{t-\tau}^{t-\alpha\tau}x(s)ds\end{bmatrix}^T\begin{bmatrix}-T_2 & T_2\\ * & -T_2\end{bmatrix}\begin{bmatrix}(1-\alpha)\tau x(t)\\ \int_{t-\tau}^{t-\alpha\tau}x(s)ds\end{bmatrix} \tag{14}$$

Here, we will consider the time-derivative of V for two cases, $0\leq\tau(t)\leq\alpha\tau$ and $\alpha\tau\leq\tau(t)\leq\tau$.

Case I: $0\leq\tau(t)\leq\alpha\tau$ We can get V_1 as follow

$$\dot{V}_1=\zeta_1^T(t)\,[\Pi_1^1 P\Pi_2^T+\Pi_2 P(\Pi_1^1)^T]\,\zeta_1(t) \tag{15}$$

From Eq.(11), by use of Eq.(5) in Lemma 2.2, we can get

$$\begin{aligned}&-\alpha\tau\int_{t-\alpha\tau}^{t}\eta^T(s)\Psi_1\eta(s)ds=\\&-\alpha\tau\int_{t-\tau(t)}^{t}\eta^T(s)\Psi_1\eta(s)ds-\alpha\tau\int_{t-\alpha\tau}^{t-\tau(t)}\eta^T(s)\Psi_1\eta(s)ds\\&\leq-\frac{\alpha\tau}{\tau(t)}\int_{t-\tau(t)}^{t}\eta^T(s)ds\Psi_1\int_{t-\tau(t)}^{t}\eta(s)ds\\&-\frac{\alpha\tau}{\alpha\tau-\tau(t)}\int_{t-\alpha\tau}^{t-\tau(t)}\eta^T(s)ds\Psi_1\int_{t-\alpha\tau}^{t-\tau(t)}\eta^T(s)ds\\&\leq-\begin{bmatrix}\int_{t-\tau(t)}^{t}\eta^T(s)ds\\ \int_{t-\alpha\tau}^{t-\tau(t)}\eta^T(s)ds\end{bmatrix}^T\begin{bmatrix}\Psi_1 & \Theta_1\\ * & \Psi_1\end{bmatrix}\begin{bmatrix}\int_{t-\tau(t)}^{t}\eta^T(s)ds\\ \int_{t-\alpha\tau}^{t-\tau(t)}\eta^T(s)ds\end{bmatrix}\end{aligned} \tag{16}$$

where Θ_1 is the matrix satisfying $\begin{bmatrix}\Psi_1 & \Theta_1\\ * & \Psi_1\end{bmatrix}\geq 0$. It should be noted that when $\tau(t)=0$ or $\tau(t)=\alpha\tau$, we have $\int_{t-\tau(t)}^{t}x(s)ds=0$ or $\int_{t-\alpha\tau}^{t-\tau(t)}x(s)ds=0$, respectively. Thus, Eq.(15) still holds. From (11) and (16), $\dot{V}_3$ satisfies:

$$\begin{aligned}\dot{V}_3\leq&\zeta_1^T(t)[(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_c^T+A_cW_{1,22}A_c^T)\\&+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_c^T+A_cW_{2,22}A_c^T)\\&-e_9W_{2,11}e_9^T-2e_9W_{2,12}e_3^T+2e_9W_{2,12}e_4^T-e_3W_{2,22}e_3^T\\&+2e_3W_{2,22}e_4^T-e_4W_{2,22}e_4^T-\Pi_3^1\begin{bmatrix}\Psi_1 & \Theta_1\\ * & \Psi_1\end{bmatrix}(\Pi_3^1)^T]\zeta_1(t)\end{aligned} \tag{17}$$

From (12), (13) and (14), $\dot{V}_4$ satisfies:

$$\begin{aligned}\dot{V}_4\leq&\zeta_1^T(t)[A_c(\gamma_1^2T_1+\gamma_2^2T_2)A_c^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_7^T\\&+2\alpha\tau e_1T_1e_8^T-e_7T_1e_7^T-2e_7T_1e_8^T-e_8T_1e_8^T-\\&(1-\alpha)^2\tau^2e_1T_2e_1^T+2(1-\alpha)\tau e_1T_2e_9^T-e_9T_2e_9^T]\zeta_1(t)\end{aligned} \tag{18}$$

Inspired by the work of [17], the following four zero equalities with any symmetric matrices $N_i(i=1,2,3)$, are considered:

$$\begin{aligned}&0=x^T(t)N_1x(t)-x^T(t-\tau(t))N_1x(t-\tau(t))\\&-2\int_{t-\tau(t)}^{t}x^T(s)N_1\dot{x}(s)ds\\&0=x^T(t-\tau(t))N_2x(t-\tau(t))\\&-x^T(t-\alpha\tau)N_2x(t-\alpha\tau)-2\int_{t-\alpha\tau}^{t-\tau(t)}x^T(s)N_2\dot{x}(s)ds\\&0=x^T(t-\alpha\tau)N_3x(t-\alpha\tau)-x^T(t-\tau)N_3x(t-\tau)\\&\quad-2\int_{t-\tau}^{t-\alpha\tau}x^T(s)N_3\dot{x}(s)ds\end{aligned} \tag{19}$$

By use of Eq.(10) and Eq.(19), we have

$$\begin{aligned}\dot{V}_6=&\zeta_1^T(t)[\alpha\tau(e_1Q_3e_1^T+A_cQ_4A_c^T)+(1-\alpha)\tau(e_1Q_5e_1^T+A_cQ_6A_c^T)\\&+e_1N_1e_1^T-e_2N_1e_2^T+e_2N_2e_2^T-e_3N_2e_3^T+e_3N_3e_3^T-e_4N_3e_4^T]\zeta_1(t)\\&-\int_{t-\tau(t)}^{t}[\eta^T(s)\begin{bmatrix}Q_3 & N_1\\ * & Q_4\end{bmatrix}\eta(s)]ds\\&-\int_{t-\alpha\tau}^{t-\tau(t)}[\eta^T(s)\begin{bmatrix}Q_3 & N_2\\ * & Q_4\end{bmatrix}\eta(s)]ds\\&-\int_{t-\tau}^{t-\alpha\tau}[\eta^T(s)\begin{bmatrix}Q_5 & N_3\\ * & Q_6\end{bmatrix}\eta(s)]ds\end{aligned} \tag{20}$$

Then combining Eqs.(8)-(9), (15), (17)-(18), (20) yields $\dot{V}\leq\zeta^T(t)\Phi_1^{(1)}\zeta(t)$. If $\Phi_1^{(1)}<0$ and $0\leq\tau(t)\leq\alpha\tau$, then $\dot{V}<0$, the system(1) is asymptotically stable.

Case II: $\alpha\tau\leq\tau(t)\leq\tau$ We can get $\dot{V}_1$ as follow

$$\dot{V}_1=\zeta_1^T(t)\,[\Pi_1^2 P\Pi_2^T+\Pi_2 P(\Pi_1^2)^T]\,\zeta_1(t) \tag{21}$$

From Eq.(11), by use of Eq.(5) in Lemma 2.2, we can get

$$\begin{aligned}&-(1-\alpha)\tau\int_{t-\tau}^{t-\alpha\tau}\eta^T(s)\Psi_2\eta(s)ds\\&=-(1-\alpha)\tau[\int_{t-\tau(t)}^{t-\alpha\tau}\eta^T(s)\Psi_2\eta(s)ds+\int_{t-\tau}^{t-\tau(t)}\eta^T(s)\Psi_2\eta(s)ds]\\&\leq-\frac{(1-\alpha)\tau}{\tau(t)-\alpha\tau}\int_{t-\tau(t)}^{t-\alpha\tau}\eta^T(s)ds\Psi_2\int_{t-\tau(t)}^{t-\alpha\tau}\eta(s)ds\\&-\frac{(1-\alpha)\tau}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)}\eta^T(s)ds\Psi_2\int_{t-\tau}^{t-\tau(t)}\eta^T(s)ds\\&\leq-\begin{bmatrix}\int_{t-\tau(t)}^{t-\alpha\tau}\eta^T(s)ds\\ \int_{t-\tau}^{t-\tau(t)}\eta^T(s)ds\end{bmatrix}^T\begin{bmatrix}\Psi_2 & \Theta_2\\ * & \Psi_2\end{bmatrix}\begin{bmatrix}\int_{t-\tau(t)}^{t-\alpha\tau}\eta^T(s)ds\\ \int_{t-\tau}^{t-\tau(t)}\eta^T(s)ds\end{bmatrix}\end{aligned} \tag{22}$$

where Θ_2 is the matrix satisfying $\begin{bmatrix}\Psi_2 & \Theta_2 \\ * & \Psi_2\end{bmatrix} \ge 0$. It should be noted that when $\tau(t) = \alpha\tau$ or $\tau(t) = \tau$, we have $\int_{t-\tau(t)}^{t-\alpha\tau} x(s)ds = 0$ or $\int_{t-\tau}^{t-\tau(t)} x(s)ds = 0$, respectively. Thus, Eq.(22) still holds. From (11) and (22), $\dot{V}_3$ satisfies:

$$\begin{aligned}\dot{V}_3 \le \zeta_1^T(t)[&(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_c^T+A_cW_{1,22}A_c^T)\\ &+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_c^T+A_cW_{2,22}A_c^T)\\ &-e_7W_{1,11}e_7^T-2e_7W_{1,12}e_1^T+2e_7W_{1,12}e_3^T-e_1W_{1,22}e_1^T\\ &+2e_1W_{1,22}e_3^T-e_3W_{1,22}e_3^T-\Pi_3^2\begin{bmatrix}\Psi_2 & \Theta_2\\ * & \Psi_2\end{bmatrix}(\Pi_3^2)^T]\zeta_1(t)\end{aligned} \tag{23}$$

From (12), (13) and (14), $\dot{V}_4$ satisfies:

$$\begin{aligned}\dot{V}_4 \le \zeta_1^T(t)[&A_c(\gamma_1^2T_1+\gamma_2^2T_2)A_c^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_7^T\\ &-(1-\alpha)^2\tau^2e_1T_2e_1^T+2(1-\alpha)\tau(e_1T_2e_8^T+e_1T_2e_9^T)\\ &-e_7T_1e_7^T-e_8T_2e_8^T-2e_8T_2e_9^T-e_9T_2e_9^T]\zeta_1(t)\end{aligned} \tag{24}$$

Inspired by the work of [17], the following four zero equalities with any symmetric matrices $N_l (l = 4,5,6)$, are considered:

$$\begin{aligned}0&=x^T(t)N_4x(t)-x^T(t-\alpha\tau)N_4x(t-\alpha\tau)\\ &\quad-2\int_{t-\alpha\tau}^{t} x^T(s)N_4\dot{x}(s)ds\\ 0&=x^T(t-\alpha\tau)N_5x(t-\alpha\tau)-x^T(t-\tau(t))N_5x(t-\tau(t))\\ &\quad-2\int_{t-\tau(t)}^{t-\alpha\tau} x^T(s)N_5\dot{x}(s)ds\\ 0&=x^T(t-\tau(t))N_6x(t-\tau(t))-x^T(t-\tau)N_6x(t-\tau)\\ &\quad-2\int_{t-\tau}^{t-\tau(t)} x^T(s)N_6\dot{x}(s)ds\end{aligned} \tag{25}$$

By use of Eq.(10) and Eq.(25), we have

$$\begin{aligned}\dot{V}_6=\zeta_1^T(t)[&\alpha\tau(e_1Q_3e_1^T+A_cQ_4A_c^T)+(1-\alpha)\tau(e_1Q_5e_1^T+A_cQ_6A_c^T)\\ &+e_1N_4e_1^T-e_2N_5e_2^T+e_2N_6e_2^T-e_3N_4e_3^T+e_3N_5e_3^T-e_4N_6e_4^T]\zeta_1(t)\\ &-\int_{t-\tau(t)}^{t}[\eta^T(s)\begin{bmatrix}Q_3 & N_4\\ * & Q_4\end{bmatrix}\eta(s)]ds\\ &-\int_{t-\alpha\tau}^{t-\tau(t)}[\eta^T(s)\begin{bmatrix}Q_5 & N_5\\ * & Q_6\end{bmatrix}\eta(s)]ds\\ &-\int_{t-\tau}^{t-\alpha\tau}[\eta^T(s)\begin{bmatrix}Q_5 & N_6\\ * & Q_6\end{bmatrix}\eta(s)]ds\end{aligned} \tag{26}$$

Then combining Eqs.(8)-(9), (21), (23)-(24), (26) yields $\dot{V} \le \zeta^T(t)\Phi_1^{(2)}\zeta(t)$. If $\Phi_1^{(2)} < 0$ and $\alpha\tau \le \tau(t) \le \tau$, then $\dot{V} < 0$, the system(1) is asymptotically stable. Thus, the proof is completed.

Remark 3.2: In order to reduce the conservatism, a new delay-dependent stability criterion is obtained in Theorem 3.1 by constructing a new Lyapunov-Krasovskii functional. In Eq.(6), V_2, V_3 and V_4 are constructed by using such an idea that the whole delay interval $[-\tau, 0]$ is decomposed into two partitions. We consider the time-varying delay $\tau(t)$ in each partition, then on each partition we choose different weighting matrices, which yields less conservative delay-dependent stability criteria, and that will be illustrated through the examples in the next section.

Remark 3.3: Recently, the reciprocally convex optimization technique was proposed in [15] and [17] to reduce the conservatism of stability criteria for systems with time-varying delays. Motivated by this work, the proposed methods of [15] and [17] were applied to the delay-decomposition method as shown in Eq. (15) and (20).

In many cases, the information on the delay derivative may not be available. Considering this case, the following result can be obtained from Theorem 3.1 by omitting V_5.

Corollary 3.4: For given scalars τ and $0 < \alpha < 1$, the system (1) with (2) is asymptotically stable if there exist positive definite matrices $P = [P_{ij}]_{5\times5}$, $\Omega_1 = [Q_{1,ij}]_{2\times2}$, $\Omega_2 = [Q_{2,ij}]_{2\times2}$, $\Psi_1 = [W_{1,ij}]_{2\times2}$, $\Psi_2 = [W_{2,ij}]_{2\times2}$, T_j, $Q_i (i = 3,4,5,6)$, any matrices $\Theta_1 = [S_{1,ij}]_{2\times2}$, $\Theta_2 = [S_{2,ij}]_{2\times2}$, $N_i (i = 1,2,3,4,5,6)$, with appropriate dimensions such that the following LMIs hold:

$$\Phi_2^{(k)}<0,\quad \begin{bmatrix}\Psi_k & \Theta_k\\ * & \Psi_k\end{bmatrix}\ge 0,\ k=1,2$$

$$\begin{bmatrix}Q_3 & N_1\\ * & Q_4\end{bmatrix}>0,\ \begin{bmatrix}Q_3 & N_2\\ * & Q_4\end{bmatrix}>0,\ \begin{bmatrix}Q_5 & N_3\\ * & Q_6\end{bmatrix}>0,$$

$$\begin{bmatrix}Q_3 & N_4\\ * & Q_4\end{bmatrix}>0,\ \begin{bmatrix}Q_5 & N_5\\ * & Q_6\end{bmatrix}>0,\ \begin{bmatrix}Q_5 & N_6\\ * & Q_6\end{bmatrix}>0,$$

where

$$\begin{aligned}\Phi_2^{(1)}=&\Pi_1^1P\Pi_2^T+\Pi_2P(\Pi_1^1)^T+(e_1Q_{1,11}e_1^T+2e_1Q_{1,12}A_c^T\\ &+A_cQ_{1,22}A_c^T)+(e_3Q_{2,11}e_3^T+2e_3Q_{2,12}e_5^T+e_5Q_{2,22}e_5^T)\\ &-(e_4Q_{2,11}e_4^T+2e_4Q_{2,12}e_6^T+e_6Q_{2,22}e_6^T)\\ &+(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_c^T+A_cW_{1,22}A_c^T)\\ &+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_c^T+A_cW_{2,22}A_c^T)\\ &-e_9W_{2,11}e_9^T-2e_9W_{2,12}e_3^T+2e_9W_{2,12}e_4^T-e_3W_{2,22}e_3^T\\ &+2e_3W_{2,22}e_4^T-e_4W_{2,22}e_4^T-\Pi_3^1\begin{bmatrix}\Psi_1 & \Theta_1\\ * & \Psi_1\end{bmatrix}(\Pi_3^1)^T\\ &+A_c(\gamma_1^2T_1+\gamma_2^2T_2)A_c^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_7^T\\ &-e_7T_1e_7^T+e_1Q_0e_1^T-(1-u)e_2Q_0e_2^T\\ &-e_7T_1e_7^T-2e_7T_1e_8^T-e_8T_1e_8^T-(1-\alpha)^2\tau^2e_1T_2e_1^T\end{aligned}$$

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$$
\begin{aligned}
&+2(1-\alpha)\tau e_1T_2e_9^T-e_9T_2e_9^T+\alpha\tau(e_1Q_3e_1^T+A_cQ_4A_c^T)\\
&+(1-\alpha)\tau(e_1Q_5e_1^T+A_cQ_6A_c^T)+e_1N_1e_1^T\\
&-(e_3Q_{1,11}e_3^T+2e_3Q_{1,12}e_5^T+e_5Q_{1,22}e_5^T)\\
&-e_2N_1e_2^T+e_2N_2e_2^T-e_3N_2e_3^T+e_3N_3e_3^T-e_4N_3e_4^T
\end{aligned}
$$

$$
\begin{aligned}
\Phi_2^{(2)}=&\Pi_1^2P\Pi_2^T+\Pi_2P(\Pi_1^2)^T+(e_1Q_{1,11}e_1^T+2e_1Q_{1,12}A_c^T\\
&+A_cQ_{1,22}A_c^T)-e_7W_{1,11}e_7^T-2e_7W_{1,12}e_1^T\\
&+(e_3Q_{2,11}e_3^T+2e_3Q_{2,12}e_5^T+e_5Q_{2,22}e_5^T)\\
&-(e_4Q_{2,11}e_4^T+2e_4Q_{2,12}e_6^T+e_6Q_{2,22}e_6^T)\\
&+(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_c^T+A_cW_{1,22}A_c^T)\\
&+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_c^T+A_cW_{2,22}A_c^T)\\
&+2e_7W_{1,12}e_3^T-e_1W_{1,22}e_1^T+2e_1W_{1,22}e_3^T-e_3W_{1,22}e_3^T\\
&-e_7T_1e_7^T+e_1Q_0e_1^T-(1-u)e_2Q_0e_2^T-\Pi_3^2\begin{bmatrix}\Psi_2 & \Theta_2\\ * & \Psi_2\end{bmatrix}(\Pi_3^2)^T\\
&+A_c(\gamma_1^2T_1+\gamma_2^2T_2)A_c^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_7^T\\
&-(1-\alpha)^2\tau^2e_1T_2e_1^T+2(1-\alpha)\tau(e_1T_2e_8^T+e_1T_2e_9^T)\\
&+\alpha\tau(e_1Q_3e_1^T+A_cQ_4A_c^T)+(1-\alpha)\tau(e_1Q_5e_1^T+A_cQ_6A_c^T)\\
&-(e_3Q_{1,11}e_3^T+2e_3Q_{1,12}e_5^T+e_5Q_{1,22}e_5^T)\\
&+e_1N_4e_1^T-e_2N_5e_2^T+e_2N_6e_2^T-e_3N_4e_3^T+e_3N_5e_3^T\\
&-e_4N_6e_4^T-e_8T_2e_8^T-2e_8T_2e_9^T-e_9T_2e_9^T
\end{aligned}
$$

Proof. The proof of this corollary immediately follows form Theorem 3.1.

When $\tau(t)$ is constant: $\tau(t)\equiv\tau$, we have the following theorem.

Theorem3.5 : For given scalars τ and $0<\alpha<1$, the system (1) with (2) is asymptotically stable if there exist positive definite matrices $P=[P_{ij}]_{5\times 5}$, $\Omega_1=[Q_{1,ij}]_{2\times 2}$, $\Omega_2=[Q_{2,ij}]_{2\times 2}$, $\Psi_1=[W_{1,ij}]_{2\times 2}$, $\Psi_2=[W_{2,ij}]_{2\times 2}$, T_j, $Q_i(i=3,4,5,6)$, any matrices $N_i(i=1,2)$, with appropriate dimensions such that the following LMIs hold:

$$\Phi_3<0,\ \begin{bmatrix}Q_3 & N_1\\ * & Q_4\end{bmatrix}>0,\ \begin{bmatrix}Q_5 & N_2\\ * & Q_6\end{bmatrix}>0,\forall j,k=1,2$$

where

$$
\begin{aligned}
\Phi_3=&\Pi_0^1P(\Pi_0^2)^T+\Pi_0^2P(\Pi_0^1)^T\\
&+(e_1Q_{1,11}e_1^T+2e_1Q_{1,12}A_{c0}^T+A_{c0}Q_{1,22}A_{c0}^T)\\
&+(e_2Q_{2,11}e_2^T+2e_2Q_{2,12}e_4^T+e_4Q_{2,22}e_4^T)\\
&-(e_3Q_{2,11}e_3^T+2e_3Q_{2,12}e_5^T+e_5Q_{2,22}e_5^T)\\
&+(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_{c0}^T+A_{c0}W_{1,22}A_{c0}^T)\\
&-(e_2Q_{1,11}e_2^T+2e_2Q_{1,12}e_4^T+e_4Q_{1,22}e_4^T)\\
&-e_6W_{1,11}e_6^T-2e_6W_{1,12}e_1^T+2e_6W_{1,12}e_2^T\\
&-e_1W_{1,22}e_1^T+2e_1W_{1,22}e_2^T-e_2W_{1,22}e_2^T\\
&-e_7W_{2,11}e_7^T-2e_7W_{2,12}e_2^T+2e_7W_{2,12}e_3^T\\
&-e_2W_{2,22}e_2^T+2e_2W_{2,22}e_3^T-e_3W_{2,22}e_3^T\\
&+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_{c0}^T+A_{c0}W_{2,22}A_{c0}^T)\\
&+\alpha\tau(e_1Q_3e_1^T+A_{c0}Q_4A_{c0}^T)+e_1N_1e_1^T-e_2N_1e_2^T\\
&-e_3N_2e_3^T+(1-\alpha)\tau(e_1Q_5e_1^T+A_{c0}Q_6A_{c0}^T)\\
&-e_6T_1e_6^T+A_{c0}(\gamma_1^2T_1+\gamma_2^2T_2)A_{c0}^T+e_2N_2e_2^T\\
&-e_7T_2e_7^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_6^T\\
&-(1-\alpha)^2\tau^2e_1T_2e_1^T+2(1-\alpha)\tau e_1T_2e_7^T
\end{aligned}
$$

Proof: Let us consider the following candidate for the appropriate Lyapunov-Krasovskii functional:

$$V=\sum_{i=1}^{6}V_i,$$

where

$$
\begin{aligned}
V_1=&\varepsilon^T(t)P\varepsilon(t),\\
V_2=&\int_{t-\alpha\tau}^{t}\eta^T(t)\Omega_1\eta(t)ds+\int_{t-\tau}^{t-\alpha\tau}\eta^T(t)\Omega_2\eta(t)ds,\\
V_3=&\alpha\tau\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}\eta^T(t)\Psi_1\eta(t)dsd\theta\\
&+(1-\alpha)\tau\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}\eta^T(t)\Psi_2\eta(t)dsd\theta,\\
V_4=&\gamma_1\int_{-\alpha\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsd\lambda d\theta\\
&+\gamma_2\int_{-\tau}^{-\alpha\tau}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsd\lambda d\theta,\\
V_5=&\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}[x^T(s)Q_3x(s)+\dot{x}^T(s)Q_4\dot{x}(s)]dsd\theta\\
&+\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}[x^T(s)Q_5x(s)+\dot{x}^T(s)Q_6\dot{x}(s)]dsd\theta,\\
\gamma_1=&\frac{(\alpha\tau)^2}{2},\ \gamma_2=\frac{(1-\alpha)^2\tau^2}{2}
\end{aligned}
$$

From V_1,V_2 ,we have their time-derivatives as:

$$\dot{V}_1=2\varepsilon^T(t)P\dot{\varepsilon}(t)=\zeta_0^T(t)\left[\Pi_0^1P(\Pi_0^2)^T+\Pi_0^2P(\Pi_0^1)^T\right]\zeta_0(t) \tag{27}$$

$$
\begin{aligned}
\dot{V}_2=&\eta^T(t)\Omega_1\eta(t)-\eta^T(t-\alpha\tau)\Omega_1\eta(t-\alpha\tau)\\
&+\eta^T(t-\alpha\tau)\Omega_2\eta(t-\alpha\tau)-\eta^T(t-\tau)\Omega_2\eta(t-\tau)\\
=&\zeta_0^T(t)[(e_1Q_{1,11}e_1^T+2e_1Q_{1,12}A_{c0}^T+A_{c0}Q_{1,22}A_{c0}^T)\\
&-(e_2Q_{1,11}e_2^T+2e_2Q_{1,12}e_4^T+e_4Q_{1,22}e_4^T)\\
&+(e_2Q_{2,11}e_2^T+2e_2Q_{2,12}e_4^T+e_4Q_{2,22}e_4^T)\\
&-(e_3Q_{2,11}e_3^T+2e_3Q_{2,12}e_5^T+e_5Q_{2,22}e_5^T)]\zeta_0(t)
\end{aligned}
\tag{28}
$$

By Eq.(4) in Lemma 2.1, we can obtain $\dot{V}_3$ as follows

$$
\begin{aligned}
\dot{V}_3=&(\alpha\tau)^2\eta^T(t)\Psi_1\eta(t)+(1-\alpha)^2\tau^2\eta^T(t)\Psi_2\eta(t) \qquad (29)\\
&-\alpha\tau\int_{t-\alpha\tau}^{t}\eta^T(s)\Psi_1\eta(s)ds-(1-\alpha)\tau\int_{t-\tau}^{t-\alpha\tau}\eta^T(s)\Psi_2\eta(s)ds\\
\le&(\alpha\tau)^2\eta^T(t)\Psi_1\eta(t)+(1-\alpha)^2\tau^2\eta^T(t)\Psi_2\eta(t)\\
&-\int_{t-\alpha\tau}^{t}\eta^T(s)ds\Psi_1\int_{t-\alpha\tau}^{t}\eta(s)ds-\int_{t-\tau}^{t-\alpha\tau}\eta^T(s)ds\Psi_2\int_{t-\tau}^{t-\alpha\tau}\eta^T(s)ds\\
=&\zeta_0^T(t)[(\alpha\tau)^2(e_1W_{1,11}e_1^T+2e_1W_{1,12}A_{c0}^T+A_{c0}W_{1,22}A_{c0}^T)
\end{aligned}
$$

$$+(1-\alpha)^2\tau^2(e_1W_{2,11}e_1^T+2e_1W_{2,12}A_{c0}^T+A_{c0}W_{2,22}A_{c0}^T)$$
$$-e_6W_{1,11}e_6^T-2e_6W_{1,12}e_1^T+2e_6W_{1,12}e_2^T$$
$$-e_1W_{1,22}e_1^T+2e_1W_{1,22}e_2^T-e_2W_{1,22}e_2^T$$
$$-e_7W_{2,11}e_7^T-2e_7W_{2,12}e_2^T+2e_7W_{2,12}e_3^T$$
$$-e_2W_{2,22}e_2^T+2e_2W_{2,22}e_3^T-e_3W_{2,22}e_3^T]\zeta_0(t)$$

Also, we can get $\dot{V}_4$ as follows:

$$\dot{V}_4=\gamma_1^2\dot{x}^T(t)T_1\dot{x}(t)+\gamma_2^2\dot{x}^T(t)T_2\dot{x}(t)$$
$$-\gamma_1\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsd\theta$$
$$-\gamma_2\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsd\theta \tag{30}$$

and

$$-\gamma_1\int_{-\alpha\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsd\theta\le$$
$$\begin{bmatrix}\alpha\tau x(t)\\ \int_{t-\alpha\tau}^{t}x(s)ds\end{bmatrix}^T\begin{bmatrix}-T_1 & T_1\\ * & -T_1\end{bmatrix}\begin{bmatrix}\alpha\tau x(t)\\ \int_{t-\alpha\tau}^{t}x(s)ds\end{bmatrix}$$
$$-\gamma_2\int_{-\tau}^{-\alpha\tau}\int_{t+\theta}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsd\theta\le$$
$$\begin{bmatrix}(1-\alpha)\tau x(t)\\ \int_{t-\tau}^{t-\alpha\tau}x(s)ds\end{bmatrix}^T\begin{bmatrix}-T_2 & T_2\\ * & -T_2\end{bmatrix}\begin{bmatrix}(1-\alpha)\tau x(t)\\ \int_{t-\tau}^{t-\alpha\tau}x(s)ds\end{bmatrix}$$

then

$$\dot{V}_4\le\zeta_0^T(t)[A_{c0}(\gamma_1^2T_1+\gamma_2^2T_2)A_{c0}^T-(\alpha\tau)^2e_1T_1e_1^T+2\alpha\tau e_1T_1e_6^T-e_6T_1e_6^T$$
$$-(1-\alpha)^2\tau^2e_1T_2e_1^T+2(1-\alpha)\tau e_1T_2e_7^T-e_7T_2e_7^T]\zeta_0(t) \tag{31}$$

From V_5 we can obtain

$$\dot{V}_5\le\zeta_0^T(t)[\alpha\tau(e_1Q_3e_1^T+A_{c0}Q_4A_{c0}^T)$$
$$+(1-\alpha)\tau(e_1Q_5e_1^T+A_{c0}Q_6A_{c0}^T)]\zeta_0(t)$$
$$-\int_{t-\alpha\tau}^{t}[x^T(s)Q_3x(s)+\dot{x}^T(s)Q_4\dot{x}(s)]ds$$
$$-\int_{t-\tau}^{t-\alpha\tau}[x^T(s)Q_5x(s)+\dot{x}^T(s)Q_6\dot{x}(s)]ds \tag{32}$$

Inspired by the work of [17], the following four zero equalities with any symmetric matrices $N_i(i=1,2)$, are considered:

$$0=x^T(t)N_1x(t)-x^T(t-\alpha\tau)N_1x(t-\alpha\tau)$$
$$-2\int_{t-\alpha\tau}^{t}x^T(s)N_1\dot{x}(s)ds$$
$$0=x^T(t-\alpha\tau)N_2x(t-\alpha\tau)-x^T(t-\tau)N_2x(t-\tau)$$
$$-2\int_{t-\tau}^{t-\alpha\tau}x^T(s)N_2\dot{x}(s)ds \tag{33}$$

By use of Eq.(32) and Eq.(33), we have

$$\dot{V}_5\le\zeta_0^T(t)[\alpha\tau(e_1Q_3e_1^T+A_{c0}Q_4A_{c0}^T)$$
$$+(1-\alpha)\tau(e_1Q_5e_1^T+A_{c0}Q_6A_{c0}^T)+e_1N_1e_1^T$$
$$-e_2N_1e_2^T+e_2N_2e_2^T-e_3N_2e_3^T]\zeta_0(t)$$
$$-\int_{t-\alpha\tau}^{t}[\eta^T(s)\begin{bmatrix}Q_3 & N_1\\ * & Q_4\end{bmatrix}\eta(s)]ds$$
$$-\int_{t-\tau}^{t-\alpha\tau}[\eta^T(s)\begin{bmatrix}Q_5 & N_2\\ * & Q_6\end{bmatrix}\eta(s)]ds \tag{34}$$

Then combining Eqs.(27)-(29), (31) and (34) yields $\dot{V}\le\zeta_0^T(t)\Phi_3\zeta_0(t)$. If $\Phi_3<0$ then $\dot{V}<0$, the system(1) is asymptotically stable, which completes the proof.

4. Numerical examples

In this section, we provide two examples to show the less conservativeness of the proposed new stability criteria in this paper.

Example 1. Consider the following neutral time-delay system $\dot{x}(t)=Ax(t)+A_1x(t-\tau(t))$ with

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\ A_1=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}$$

When $\dot{\tau}\le\mu<1$, applying Theorem 3.1, the corresponding maximum admissible upper bounds are given in Table 1 which clearly shows that the effectiveness of the delay-decomposition approach.

Example 2. Consider the following nominal neutral system with constant time-delay

$$\dot{x}(t)-\begin{bmatrix}-0.2 & 0\\ 0.2 & -0.1\end{bmatrix}\dot{x}(t-\tau)$$
$$=\begin{bmatrix}-0.9 & 0.2\\ 0.1 & -0.9\end{bmatrix}x(t)+\begin{bmatrix}-1.1 & -0.2\\ -0.1 & -1.1\end{bmatrix}x(t-\tau)$$

For above system, the maximum delay bounds for asymptotic stability were investigated in [25], [26], [27] and [16]. From Table 2, it can be seen that the obtained delay bounds by Theorem 3.5 are larger than those of [25], [26], [27] and [16].

5. Conclusion

It this paper, new delay-dependent stability criteria for neutral time-delay systems are proposed. In order to obtain less conservative results, a new delay-decomposition method is used to improve the maximum admissible upper bounds of stability criterion. Numerical examples have been given to show that our stability are less conservative than some existing ones in the literatures.

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Table 1: The maximum admissible upper bounds of time-varying delays with different values of μ (Example1)

μ	**0**	**0.1**	**0.5**	**0.9**	**unknown**
[18]	4.47	3.60	2.00	1.18	-
[13]	4.47	3.60	2.00	1.18	-
[19]	4.47	3.60	2.00	1.18	-
[20]	4.47	3.60	2.04	1.37	-
[21]	4.47	3.66	2.33	1.87	-
[22]	5.55	4.41	2.40	2.12	2.12
Theorem 3.1	5.87 ($\alpha = 0.59$)	4.43 ($\alpha = 0.46$)	2.46 ($\alpha = 0.22$)	2.22 ($\alpha = 0.51$)	2.22 ($\alpha = 0.51$)

Table 2: The maximum admissble upper bounds of constant time delays (Example2)

Method	τ
[25]	1.8037
[26]	1.9132
[27]	2.0054
[16]	2.1046
Theorem 3.5	2.1445 $\alpha = 0.57$