Formal Validation for Software Modeling

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Abstract
Currently, modeling for software is mostly semiformal, such as UML (Unified Modeling Language). The main problem is difficult to analyze semantics and verify correctness for a vital system. CPN (Colored Petri Net) as modeling and verification method provides formal semantics and a number of analysis techniques and tools. This paper presents an approach of formal modeling and validation for software process, which transforms UML models based on RUP (Rational Unified Process) to CPN and uses CPN tools to investigate the behavior of modeled system.

Keywords: Modeling, CPN, Formal, UML.

1. Introduction
UML as a semiformal modeling language is widely applied to many large and complex software systems. Although UML’s grammar is formal, natural language is used by its semantics, which makes it only to do static modeling not to formally analyze and verify the models. Formal methods (e.g. CPN) that have precise mathematical semantics and automated verification tools are used for both static structural analysis and dynamic behavior analysis. CPN as a graphically oriented modeling language has functions of ordinary Petri Nets and high level programming languages providing the primitives for the definition of data types and describing data manipulation, which makes it suitable for modeling for complex systems. The CPN language allows the model to be represented as a set of modules, which can make it to be organized as a set of hierarchically related modules, i.e. HCPN (Hierarchical Coloured Petri Net). CPN model is one of best ways for analysis and design of non-deterministic and concurrent systems. In the paper, I present a detailed statement of applying CPN to describe the semantics of UML diagram, and also do analysis and verification of object-oriented system[4].

2. Related Theory Knowledge
2.1 Formal Definition of HCPN
HCPN makes it possible to relate a amount of individual CPN to each other in a formal way. This is similar to the situation in which a programmer constructs a large-scale program from a set of modules and subroutines[8].

A Hierarchical Coloured Petri Net is a tuple $HCPN=(S, SN, SA, PN, PT, PA, FS, PT, PP)$ satisfying the following requirements below[1][2]:

(i) $S$ is a finite set of pages such that:

- Each page $s \in S$ is a non-hierarchical CPN:
- $(\Sigma s, Ps, Ts, As, Ns, Cs, Gs, Es, Is )$.
- The sets of net elements are pairwise disjoint:
- $\forall s_1, s_2 \in S : s_1 \neq s_2 \Rightarrow (Ps_1 \cup Ts_1 \cup As_1) \cap (Ps_2 \cup Ts_2 \cup As_2) = \emptyset$.

(ii) $SN \subseteq T$ is a set of substitution nodes.

(iii) $SA$ is a page assignment function. It is defined from $SN$ into $S$ such that:

$\forall s_1, s_2 \in S : s_1 \neq s_2 \Rightarrow (Ps_1 \cup Ts_1 \cup As_1) \cap (Ps_2 \cup Ts_2 \cup As_2) = \emptyset$.

(iv) $PT \subseteq P$ is a set of port nodes.

(v) $PT$ is a port type function. It is defined from $PN$ into { in, out, i/o, general }.

(vi) $PA$ is a port assignment function. It is defined from $SN$ into binary relations such that:

- $PA(t)$ is a set of port nodes:
- $\forall t \in SN: PA(t) \subseteq X(t) \times PNSA(t)$.
- Socket nodes are related to port nodes: $\forall t \in SN: PA(t) \subseteq X(t) \times PNSA(t)$.
- Socket nodes are of the correct type:
- $\forall t \in SN: \forall (p_1, p_2) \in PA(t): [PT(p_2 \neq general ST(p_1, t) = PT(p_2)]$. 

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Related nodes have identical colour sets and equivalent initialization expressions:

\[ \forall t \in S \forall (p_1, p_2) \in \text{PA}(t): \left[ C(p_1) = C(p_2) \land \text{Al}(p_1) < > = \text{I}(p_2) < > \right]. \]

(vii) \( \text{FS} \subseteq S_p \) is a finite set of fusion sets such that:
Members of a fusion set have identical colour sets and equivalent initialization expressions:

\[ \forall s \in \text{FS}: \forall (p_1, p_2) \in \text{fs}: \left[ C(p_1) = C(p_2) \land \text{Al}(p_1) < > = \text{I}(p_2) < > \right]. \]

(viii) \( \text{FT} \) is a fusion type function. It is defined from fusion sets into \{global, page, instance\} such that:
Page and instance fusion sets belong to a single page:

\[ \forall s \in \text{FS}: \left[ \text{FT}(s) \# \text{global} \right] \exists s \in S: \text{fs} \subseteq S_p. \]

(ix) \( \text{PP} \in S_{MS} \) is a multi-set of prime pages.

2.2 Dynamic properties of coloured petri net

When it is represented by a CPN, UML models can be analyzed and verified by using dynamic properties.

(1) Boundedness properties [1]
Definition 1: Let a set of token elements \( X \subseteq \text{TE} \) (we use \( \text{TE}(p) \) to denote the set of token elements that correspond to a given place \( p \)) and a non-negative integer \( n \in \mathbb{N} \) be given:

(i) \( n \) is an upper bound for \( X \) iff:

\[ \forall M \ni \text{M}^{0>}: \left| (M|X) \right| \leq n. \]

(ii) \( n \) is a lower bound for \( X \) iff:

\[ \forall M \ni \text{M}^{0>}: \left| (M|X) \right| \geq n. \]

The set \( X \) is bounded iff it has an upper bound.

(2) Liveness Properties [1]
Definition 2: Let a marking \( M \in M \) and a set of binding elements \( X \subseteq \text{BE} \) (we use \( \text{BE}(t) \) to denote the set containing all those binding elements which correspond to a given transition \( t \)) be given:

(i) \( M \) is dead iff no binding element is enabled, i.e., iff:

\[ \forall x \in \text{BE}: \text{a} \neq M^{x>}. \]

(ii) \( M \) is dead in \( M \) iff no element of \( X \) can become enabled, i.e., iff:

\[ \forall M' \ni M^{0>}: \forall x \in X: \text{a} \neq M'^{x>}. \]

(iii) \( X \) is live iff there is no reachable marking in which it is dead, i.e., iff:

\[ \forall M' \ni M^{0>}: \exists M'' \ni M^{0>}: \exists x \in X: M''^{x>}. \]

(3) Fairness Properties [1]

We use \( \text{EN}(\sigma) = \sum_{k \in N} \text{EN}_{\text{X}}(\sigma)_k \), \( \text{OC}(\sigma) = \sum_{k \in N} \text{OC}_{\text{X}}i(\sigma)_k \)

Definition 3: Let \( X \subseteq \text{BE} \) be a set of binding elements and \( \sigma \in \text{OSI} \) an infinite occurrence sequence [1].

(i) \( X \) is impartial for \( \sigma \) iff it has infinitely many occurrences, i.e., iff:

\[ \text{OC}(\sigma) = \infty. \]

(ii) \( X \) is fair for \( \sigma \) iff an infinite number of enablings implies an infinite number of occurrences, i.e., iff:

\[ \text{EN}(\sigma) = \infty. \]

(iii) \( X \) is just for \( \sigma \) iff a persistent enabling implies an occurrence, i.e., iff:

\[ \forall i \in N+: \left[ \text{EN}_{\text{X}}(\sigma)_i \neq 0 \right] \exists k \geq i: \left[ \text{EN}_{\text{X}}(\sigma)_k = 0 \right]. \]

\[ \forall \text{OC}_{\text{X}}(\sigma) = 0]. \]

(4) Home Properties [1]
Definition 4: Let a marking \( M \in M \) and a set of marking \( X \subseteq M \) be given:

(i) \( M \) is a home marking iff \( \forall M' \ni M^{0>}: M' \neq M^{> \Phi}. \)

(ii) \( X \) is a home space iff \( \forall M' \ni [M^{0}> X']\ni M' \neq M^{> \Phi}. \)

We use HOME to denote the set of all home markings.

3. A Formal Approach For UML Diagram

A UML class diagram (includes two kinds of classes, i.e. general class and association class), which describes static structure and is a basis of system implementing. A UML dynamic diagram (e.g. activity diagram, sequence diagram) describing behaviour and characteristic must be based on the class diagram. So, UML class diagram plays a key role in the system modeling. However, UML is a semiformal modeling language, and it is difficult to formally analyze and verify the models. While CPN has a graphical representation, a number of modeling techniques and a well-defined semantics allowing formal analysis. Obviously, for obtaining increased confidence in the correctness of the model, it is necessary to transform UML diagram to CPN model.

Before transforming, two works must be done [9]:

- A association class between two classes need be created, which only includes attributes. The attributes are got from UML activity diagram.
- A general class including attributes and operations need be transformed into a whole class that only includes operations and one or more part classes that only include attributes.

Below, by using Hierarchical Coloured Petri Net an algorithm of formal modeling is given:

- A substitution transition presents a class which has two and more operations. Each operation corresponds with a transition in substitution transition subpage. The socket place of the substitution transition is related with the port place in the subpage. The contents of its arc expressions are decided by the message that is transmitted between different action states in UML activity diagram.
- A general transition presents a class that only has one operation.
- A place presents a class that only has static attributes that correspond with colour sets of the place.
- The direction of a directed arc in HCPN is determined by UML class diagram.
4. Formal Analysis and Validation

4.1 Modeling Based on RUP

RUP is an iterative software development process framework (from requirement to implementation). It may be applied to a variety of different types of software systems, applications, organizations and projects. At present UML combined with RUP implements best software practices.

Now, I present an example that is a goods ordering system to explain how to formalize UML diagram and use CPN tools to verify the behaviour of the modelled system[6].

4.1.1 Requirement modeling

![Fig. 1. a use case diagram of ordering system](image)

![Fig. 2. an activity diagram of ordering system](image)

4.1.2 Design modeling

![Fig. 3. a class diagram of ordering system](image)

4.2 Formal Modeling and Validation

According to the transforming algorithm, a HCPN is given below. The colour sets and variables are described by CPN ML[5].

\[
\begin{align*}
\text{colset & Goods_Id=string;} \\
\text{colset & Goods_Amount=int;} \\
\text{colset & Supplier_Id=string;} \\
\text{colset & Goods_Name=string;} \\
\text{colset & Lack_Goods_Info=product Goods_Id*Goods_Amount*Supplier_Id;} \\
\text{colset & Supply_Goods_Info=product Goods_Id*Goods_Name*Goods_Amount;} \\
\text{colset & Buy_Goods_Info=product Goods_Id*Goods_Amount;} \\
\text{var & gid: Goods_Id;} \\
\text{var & gna:Goods_Name;} \\
\text{var & gam:Goods_Amount;} \\
\text{var & sid:Supplier_Id;}
\end{align*}
\]

![Fig. 4. the prime HCPN model of ordering system](image)

![Fig. 5. subpage correspond with substitution transition Operator_Role](image)
Full state spaces represent all possible executions of the model being analysed. The state space of a CPN model make it possible to verify, i.e., prove in the mathematical sense of the word that the model possesses a certain formally specified property[3][7].

### Boundedness Properties

<table>
<thead>
<tr>
<th>Place</th>
<th>Upper</th>
<th>Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCPN'Buy_Goods_Info</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>HCPN'Lack_Goods_Info</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>HCPN'Start</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>HCPN'Stop</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>HCPN'Supply_Goods_Info</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>W_OP'Success</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**Best Upper Multi-set Bounds**

<table>
<thead>
<tr>
<th>Place</th>
<th>Multiset Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCPN'Buy_Goods_Info</td>
<td>(&quot;0110&quot;,200)</td>
</tr>
<tr>
<td>HCPN'Lack_Goods_Info</td>
<td>(&quot;0110&quot;,200,&quot;s010&quot;)</td>
</tr>
<tr>
<td>HCPN'Start</td>
<td>e</td>
</tr>
<tr>
<td>HCPN'Stop</td>
<td>e</td>
</tr>
<tr>
<td>HCPN'Supply_Goods_Info</td>
<td>(&quot;0110&quot;,&quot;DELL&quot;,200)</td>
</tr>
<tr>
<td>W_OP'Success</td>
<td>(&quot;0110&quot;,&quot;DELL&quot;,200)</td>
</tr>
</tbody>
</table>

**Best Lower Multi-set Bounds**

<table>
<thead>
<tr>
<th>Place</th>
<th>Multiset Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCPN'Buy_Goods_Info</td>
<td>empty</td>
</tr>
<tr>
<td>HCPN'Lack_Goods_Info</td>
<td>empty</td>
</tr>
<tr>
<td>HCPN'Start</td>
<td>empty</td>
</tr>
<tr>
<td>HCPN'Stop</td>
<td>empty</td>
</tr>
<tr>
<td>HCPN'Supply_Goods_Info</td>
<td>empty</td>
</tr>
<tr>
<td>W_OP'Success</td>
<td>empty</td>
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</tbody>
</table>

### Fairness Properties

- No infinite occurrence sequences.

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The state space is calculated by CPN state tools that use a state construction algorithm. A node of the directed graph correspond to reachable marking, and an arc corresponds to occurring binding elements. The best upper(lower) integer bound of a place specifies the maximal(minimal) number of tokens residing on a place in any reachable marking. For example, the best upper integer bound of the place Buy_Goods_Info is 1 that means at most there is one token, and there exists reachable markings where there is a token on it. This is what the modeler would expect, since Buy_Goods_Info is always supposed to contain a single token with a colour corresponding to the data which has been received. The other places are similar to it. Fig. 6 shows there is a single home marking that has the node number corresponding to node 6 of fig. 6, which means that it is possible to have occurrence sequences that can be extended to reach Home Marking. The fact that node 6 is the only Dead Marking clarify that the ordering system as specified by the CPN model is correct, i.e., if execution terminates we have the correct results. Meanwhile, because node 6 is also a home marking it is always possible to end the system with the correct result. Finally, Figure 6 shows that no dead transitions exist, which indirectly tells us that UML model has no redundant methods.

### 5. Conclusion

As an important modeling approach, UML has been successfully applied in many fields of software engineering. Because of lackness of formal semantics, UML is difficult to verify the correctness of models, which may cause disastrous consequences for a vital system. CPN has precise mathematical semantics and automated verification tools. CPN combined with UML is a better modeling approach, which not only
easily describes the software but also detects errors and obtains increased confidence in the correctness of the model and thereby the system.

References