Input-Sensitive Fuzzy Cognitive Maps

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Abstract
Complex systems with nonlinearities and surrounding uncertainty are usually modeled sufficiently by Fuzzy Cognitive Maps (FCMs). FCMs work efficiently even with missing data. Experts, for each case study, support with their knowledge the developed FCMs. Nevertheless, the main drawback of FCMs is their convergence to the same equilibrium point regardless of the initial conditions. In this paper a different approach for modeling FCMs is proposed, where the inputs gain back their lost importance. Thus, Input-Sensitive Fuzzy Cognitive Maps (IS-FCMs), supported both by experts and by the appropriate Rule-Base, manage to converge to desired operating points. The Nonlinear Hebbian Learning algorithm (NHL) is used in order to optimize the values of the weights. A PV-System application is presented. The simulation results support the hypothesis of the proposed new IS-FCM model.

Keywords: Fuzzy Systems, Fuzzy Cognitive Maps, Renewable Energy Sources, Photovoltaics.

1. Introduction
The majority of real dynamic complex systems are characterized by nonlinearity, great uncertainty and noise disturbance. A modern way to simulate them is via Neural Networks (NN) and Fuzzy Cognitive Maps (FCMs). FCMs combine the robust properties of fuzzy logic and neural networks (NN). An extensive survey about FCMs has been made in [12]. Provided with the necessary knowledge-database, and supported by the appropriate experts for each system, FCMs function sufficiently even with missing data. Experts are those who define both the interconnections among the concepts (neurons) with appropriate weights and the input-output relationship. Given this defined relationship, the FCM tries to simulate the function \( y = f(x) \). Thus, the more input-output pairs the FCM is provided with, the more efficiently the abovementioned function \( f \) will be simulated. The process of defining these weights was static and experts did not take into consideration the present state of the concepts, or in other words, the initial conditions of the system.

Since FCMs are an evolution of NN, concepts should be treated like neurons. An activated neuron in human neural system behaves differently than a deactivated one. Thus, the interconnections among concepts should be depended on their initial conditions, and especially the initial conditions of the input-concepts.

In this paper an input-sensitive FCM (IS-FCM) is introduced. Due to the increasing scientific interest in Renewable Energy Sources, and in particular Photovoltaics, a relative application is presented. A PV-System for a house, which interacts with its environment, is modeled by the proposed IS-FCM. The simulation results for the PV-System are quite promising.

The paper is organized as follows: Section 2 makes a brief introduction in Fuzzy Cognitive Maps (FCMs) and repeats, for the sake of completeness, the Nonlinear Hebbian algorithm, which improves the weights defined by the experts. In section 3 the proposed input-sensitive FCM (IS-FCM) is introduced while in section 4 simulation results are presented. Finally in section 5 conclusions are briefly discussed.

2. Fuzzy Cognitive Maps

2.1 Mathematical Representation of FCMs
The graphical illustration of an FCM is a signed directed graph with feedback, consisting of nodes and weighted arcs [1]. Nodes of the graph stand for the concepts that are used to describe the behavior of the system and they are connected by signed and weighted arcs representing the causal relationships that exist between the concepts (Fig. 1).

Each concept is characterized by a number \( A_i \) that represents its value and it is calculated through the transformation of the fuzzy value of the system’s variable, for which this concept stands, in the interval \([0, 1]\). There are three possible types of causal relationships that express
the type of influence of one concept to another. The weights of the arcs between concept \( C_i \) and concept \( C_j \) could be positive \( (W_{ij} > 0) \) which means that an increase in the value of concept \( C_i \) leads to the increase of the value of concept \( C_j \), and a decrease in the value of concept \( C_i \) leads to the decrease of the value of concept \( C_j \). A negative causality \( (W_{ij} < 0) \) means that an increase in the value of concept \( C_i \) leads to the decrease of the value of concept \( C_j \) and vice versa. Finally it could be \( W_{ij}=0 \) which means that \( C_i \) and \( C_j \) are not correlated in some way. The sign of \( W_{ij} \) indicates whether the relationship between concepts \( C_i \) and \( C_j \) is direct or inverse.

![Fig. 1 The fuzzy cognitive map model.](image)

In particular, the causal interrelationships among concepts are declared using the variable Influence which is interpreted as a linguistic variable taking values in the universe of discourse \( U = [-1, 1] \). Its term set \( T(\text{influence}) \) is suggested to be comprised of eight variables. Using eight linguistic variables, an expert can describe in detail the influence of one concept on another and can discern between different degrees of influence. The nine variables used here are: \( T(\text{influence}) = \{ \text{zero, very very low, very low, low, medium, high, very high, very very high, one} \} \). The corresponding membership functions for these terms are shown in Fig. 2 and they are \( \mu_{vl}, \mu_{vlvl}, \mu_{vlh}, \mu_{lh}, \mu_{vh}, \mu_{vh}, \mu_{vh}, \mu_{vh} \) and \( \mu_0 \). A positive sign in front of the appropriate fuzzy value indicates positive causality while a negative sign indicates negative causality.

The proposed fuzzy values for the interconnections among the concepts are aggregated using the MAX method. Then, the COA defuzzification method is used to transform the overall fuzzy weight into a crisp numerical weight \( w_{ij} \), belonging to the interval \([-1, 1]\). A few defuzzification methods are discussed in [19]. A detailed description of the development of an FCM model is given in [1, 2, 3, 4, 10, 14, 15, 16].

![Fig. 2 Membership function of the linguistic variables.](image)

The value \( A_i \) of concept \( C_i \) represents the degree which corresponds to its physical value. At each simulation step, the value \( A_i \) of a concept \( C_i \) is calculated by computing the influence of the interconnected concepts \( C_j \)'s on the specific concept \( C_i \) following the recursive equation (Eq.1):

\[
A_i^{(t+1)} = f(A_i^{(t)} + \sum_{j=1}^{N} A_j^{(t)} W_{ij})
\]

, where is the value of concept \( C_i \) at simulation step, is the value of concept \( C_j \) at simulation step \( k \), is the weight of the interconnection from concept \( C_j \) to concept \( C_i \) and \( f \) is the sigmoid threshold function (Eq.2):

\[
f = \frac{1}{1 + e^{-\lambda x}}
\]

, where \( \lambda > 0 \) is a parameter determining its steepness.

There is only a brief reference to FCMs, since, further information and details about Fuzzy Cognitive Maps and their theories are analytically discussed in [2, 3, 4, 5, 12, 18]. Moreover, an interesting application of FCMs in Information Technology (IT) is presented in [17].

2.2 Nonlinear Hebbian Learning Algorithm (NHL)

The NHL [6, 7] algorithm is chosen and implemented in this paper, so as to optimize the values of the weight matrix \( W_{ij} \). At first the experts suggest a desired region in which the decision output concepts \( (DOCs) \) should move. The desired regions for the output nodes reflect the smooth and efficient operation of the modeled system, e.g.:

\[
0.72 \leq DOC_i \leq 0.83
\]
Basic factor of the NHL algorithm is the minimization of two basic cost functions (Eq.3, Eq.4) in order to have a convergence after a finite number of iteration steps [8, 9]:

\[
F_1 = J_1 = \left\| DOC_i - T \right\| < \varepsilon_1
\]

(3)

and

\[
F_2 = J_2 = \left| DOC_i^{(k)} - DOC_i^{(k-1)} \right| < \varepsilon_2
\]

(4)

where \( T \) is the hypothetic desired value of the output and it is usually the average of the defined range by the experts. Eq.3 is the square of the Euclidian norm between the target and the output concept of the FCM, and Eq.4 is the absolute difference between the last two values of the output concept at step \( k \) and \( k-1 \). Basic idea of the NHL algorithm is the adjustment of the initial (\( w_{ij}^{\text{initial}} \)) matrix weights, defined by the appropriate experts, in a way that the Decision Output Concepts converge inside the desired region (Eq.5):

\[
w_{ij}^{(k)} = \gamma \cdot w_{ij}^{(k-1)} + \\
\quad + \eta \cdot A_{ij}^{(k-1)} \cdot \left( A_{ij}^{(k-1)} - \text{sgn}(w_{ij}^{(k-1)} \cdot w_{ij}^{(k-1)} \cdot A_{ij}^{(k-1)}) \right)
\]

(5)

Each non-zero element of the final weight matrix is improved and converges into an optimal value according to the specific cost functions of the problem. An acceptable range of change could be defined for these weights. If one weight takes a value out of the desired region then the experts’ suggestions should be reconsidered, e.g.: if \( \left| w_{ij}^{\text{final}} - w_{ij}^{\text{initial}} \right| > \lambda \) (where \( \lambda \) is the error-quantity) then that means that concept \( C_i \) has a different relationship with concept \( C_j \) than the initial one defined from the experts, and their correlation should be reevaluated. Hence, the more reliable the experts are, the smaller this error quantity (\( \lambda \)) should be obtained.

The learning parameters \( \gamma, \eta \) of Eq.5 are very important and they usually take values between \( \gamma \in [0,9,1] \) and \( \eta \in [0,0,1] \) respectively. Each case study requires its own parameters \( \gamma, \eta \) and their sensitivity varies.

3. Input Sensitive FCMs (IS-FCMs)

Up to now, experts were defining an optimal weight for the correlation between two concepts. After applying appropriate penalization algorithms the optimal value for each weight was determined. So, the initial weight matrix \( W_0 \) was determined regardless of the initial conditions (inputs) of the system. Even with learning algorithms that had been developed [7], the system was always converging to the same equilibrium point without taking into consideration the inputs of the system. The whole structure of FCMs was very static and didn’t allow any interaction. Until now, experts were asked to define a relationship between two concepts regardless of their state, or else their inputs. On the other hand, an activated concept (neuron) behaves differently than a deactivated one. Hence, initial conditions are important.

In this paper a different approach is proposed, where the experts do not define an exact value but a range, in which each weight \( w_0 \) should lie, always taking into account the inputs of the system. Thus, instead of considering \( w_0 = p \) it would be better to consider \( w_0 \in [a, b] \) and take a value randomly inside this region. Besides, the response of a PV-System is not always steady. Even with the same initial conditions of insolation and temperature the PV-System might deliver different values of power, for several reasons that are beyond the scope of this paper (e.g.: PV panels failure, uncertainty, etc.).

Moreover, for each input vector, the appropriate output range should be determined by the experts. A PV-System cannot produce 10kW if it is night or a cloudy day. So, experts should be reasonable and provide the engineer with the appropriate database so the latter could construct a FCM which delivers feasible and rational results. Nevertheless, an FCM remains a simulated model and not an accurate prototype of the real system. Hence, the possibility of failure is present.

3.1 Comparison of Threshold Functions

Each of these functions has to satisfy the following criteria:

a) Continuous and differentiable
b) Finite number of upper and lower limits

The most common threshold in FCMs is the sigmoid function. Fig. 3 shows the sigmoid function for several values of “\( \gamma \)” parameter. The larger the “\( \gamma \)”, the steeper the function becomes. The following equation (Eq.2) describes the function described above:

\[
f(x) = \frac{1}{1 + e^{-\lambda x}}
\]
for $\lambda \gg 0 \Rightarrow f(x) \rightarrow \text{binary threshold function} \rightarrow $

$\Rightarrow f(x) = \frac{1}{1+e^{-\lambda x}} \Rightarrow f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

Verification of criteria satisfaction:

$f(-\infty) = -1$

$f(0) = 0$

$f(+\infty) = 1$

$f(x) = \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\lambda x} + e^{-\lambda x}} = \frac{1}{1 + e^{-2\lambda x}} - \frac{1}{1 + e^{2\lambda x}} \Rightarrow$

$\Rightarrow f(x) = \tanh(\lambda \cdot x)$ (6)

Since the abovementioned criteria a, b are satisfied, the hyperbolic tangent can be used as a threshold function (Fig. 4, Fig. 5).

3.2 PV-System case study using IS-FCMs

The main aim is to turn the PV-System into an input-sensitive one, in order to interact with its environment. In this paper a simplified model of an already developed FCM for PV-System [11] is considered. In the case of PV-Systems the problem of Maximum Power Point is crucial and should not be overlooked [13]. However, for the purpose of this paper, the Maximum Power Point Tracker (MPPT) issue is not addressed. Fig. 6 illustrates the proposed IS-FCM of the photovoltaic system. It is considered that insolation ($C_1$) and temperature ($C_2$) are the input concepts, and PV-Power ($C_3$) the output concept. These concepts take values between 0 and 1. This region is normalized and represents the lower and upper limits of the real size of each concept respectively, e.g.: regarding the insolation: $[0,1] \rightarrow [0,1000] \text{W/m}^2$, where $1000\text{W/m}^2$ is the
The Insolation concept ($C_1$) is considered to be equivalent to the insolation falling perpendicularly to each square meter ($\text{m}^2$) of the photovoltaic panel. According to the theory described above, for different insolation and temperature, different power (kW) should be delivered. So far, there was meaningless to change the inputs because the result was always the same; convergence to the very same equilibrium point. In conventional control theory this could be explained as if there was a very powerful attractor; or else an infinite domain of attraction. So, regardless of weather conditions, season, sunlight, and temperature, the conventional FCM would converge to the same operating point. On the other hand, it is impossible for a PV-System to deliver the same power at summer or winter, or at day or night. Thus, the logic of IF-THEN rules was introduced in order to improve the flexibility as well as the robustness of the FCM. Depending on the inputs, the appropriate rules were developed in order to derive the initial weight matrix $W_{ij}$ as well as to adjust the parameters $\eta$, $\gamma$ of the NHL algorithm.

The following figure (Fig. 7) shows the membership functions of the inputs $C_1, C_2 \in \{vl, l, m, h, vh\}$ (very low, low, medium, high, very high). The user can enter any input described by the abovementioned linguistic variables. Not all input combinations are considered here. Some input pairs are not possible because they may result in conditions that are either irrational or contradictory. Then the COA defuzzification method is utilized to transform the fuzzy values into crisp ones. Subsequently, the developed IS-FCM delivers the estimated photovoltaic efficiency. The experts describe the interconnection among concepts with the linguistic variables presented in Fig. 2. The twenty five (25) developed rules improved the robustness of the IS-FCM.

Since the inputs (insolation and temperature) as well as the output (PV power) take values between 0 and 1, the hyperbolic tangent function seems to be more appropriate for this energy problem. The sigmoid function is generally used for input vectors taking values between -1 and 1. By choosing the hyper-tangent as threshold, the whole region, in x axis, between 0 and 1 is fully exploited. In the particular case:

$$x = A_i^{(k)} + \sum_{j=1}^{N} A_j^{(k)} W_{ji}$$

This could be translated to:

$$f(x) = f(A_i^{(k)} + \sum_{j=1}^{N} A_j^{(k)} W_{ji}) \Rightarrow$$

$$\tanh(x) = \tanh(A_i^{(k)} + \sum_{j=1}^{N} A_j^{(k)} W_{ji}) = A_i^{(k+1)}$$

The $x$ factor in the specific problem takes values $x \in [0,1]$. Hence, a simple comparison between Fig. 3 and Fig. 4 reveals the reasons for which the hyperbolic tangent was selected.

### 4. Simulation Results

The following simulations concern the climate of Greece and particularly the Western Region of Greece (Patras). The PV-System is considered to be a house installation, known as 10kW-photovoltaic system. Six case studies for random inputs are investigated. The initial value of the output $C_3$ was randomly chosen. The results are presented in Table 1. Fig. 8 to Fig. 13 illustrate the response of the system for the particular inputs. The IS-FCM converges when the following criteria are satisfied:

$$e_i = J_i = |A_i^{(k)} - A_i^{(k-1)}| < 5 \cdot 10^{-3}$$
and

\[ e_2 = J_2 = |A_s^{(k)} - T_3| < 5 \cdot 10^{-5} \]

The error \( e_2 \) was chosen small enough so that, despite uncertainty, the output may track the target within an acceptable deviation. The final difference between the output and the target is:

\[ \| A_s^{(k)} - T_3 \| = |e_2| \approx 7.1 \cdot 10^{-3} > e_2 \]

Even then, the developed algorithm successfully forces the output (\( C_s \)) to meet the target (\( T_3 \)) after finite (\( k \)) iteration steps.

<table>
<thead>
<tr>
<th>Case</th>
<th>Inputs (fuzzy values)</th>
<th>( C_s ) (output)</th>
<th>( k ) iteration steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>( C_1 = vh ) ( C_2 = vh )</td>
<td>0.6141</td>
<td>18</td>
</tr>
<tr>
<td>2nd</td>
<td>( C_1 = sh ) ( C_2 = sh )</td>
<td>0.6482</td>
<td>13</td>
</tr>
<tr>
<td>3rd</td>
<td>( C_1 = m ) ( C_2 = h )</td>
<td>0.5358</td>
<td>6</td>
</tr>
<tr>
<td>4th</td>
<td>( C_1 = l ) ( C_2 = m )</td>
<td>0.5217</td>
<td>18</td>
</tr>
<tr>
<td>5th</td>
<td>( C_1 = l ) ( C_2 = l )</td>
<td>0.4632</td>
<td>19</td>
</tr>
<tr>
<td>6th</td>
<td>( C_1 = vl ) ( C_2 = m )</td>
<td>0.0523</td>
<td>25</td>
</tr>
</tbody>
</table>

The dashed black line illustrates the desired output target while the blue one shows how the output asymptotically converges to the desired one. After a significant number of simulations, the appropriate initial weights for each case as well as the learning parameters \( \eta \), \( \gamma \) of the NHL algorithm were determined. Five energy experts supported this research with their knowledge.

![Fig. 8 IS-FCM output response (Case_1).](image)

![Fig. 9 IS-FCM output response (Case_2).](image)

![Fig. 10 IS-FCM output response (Case_3).](image)
It is observed that the IS-FCM model is indeed turned into an input-sensitive one; e.g.: the higher the insolation is, the greater the photovoltaic efficiency becomes. Thus, the objective to make the FCM model input-sensitive is successfully accomplished. The initial and the final weight matrix only for the 1st case \((A_1^{(1)} = vh, A_2^{(1)} = vh)\) are presented below in order to show how the NHL algorithm improves the initial weights so that the output can meet the target.

\[
W_{initial} = \begin{bmatrix}
0 & 0.6337 & 0.9659 \\
0 & 0 & -0.4949 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
W_{NHL}^{(1)} = \begin{bmatrix}
0 & 0.4409 & 0.6262 \\
0 & 0 & -0.1049 \\
0 & 0 & 0
\end{bmatrix}
\]

Fig. 14 shows the minimization of the cost functions for the 1st case study \((A_1^{(1)} = vh, A_2^{(1)} = vh)\).

Both \(e_1\) and \(e_2\) converge rapidly after 18 iteration steps to values approximately around zero.

The process for the calculation of the weights follows. A specific example is presented regarding the initial weight matrix \(W_{initial}^{(1)}\) of the 1st case and particularly the element \(w_{12}\). Given that \(A_1^{(1)} = vh\) and \(A_2^{(1)} = vh\), the experts claimed the following:

1st expert:
If a small change occurs in node \(C_1\) then a very high change is caused in node \(C_2\)
Inference: Influence from concept $C_1$ to $C_2$ is positively very high

2nd expert:
If a small change occurs in node $C_1$ then a very high change is caused in node $C_2$
Inference: Influence from concept $C_1$ to $C_2$ is positively very high

3rd expert:
If a small change occurs in node $C_1$ then a high change is caused in node $C_2$
Inference: Influence from concept $C_1$ to $C_2$ is positively high

4th expert:
If a small change occurs in node $C_1$ then a medium change is caused in node $C_2$
Inference: Influence from concept $C_1$ to $C_2$ is positively medium

5th expert:
If a small change occurs in node $C_1$ then a high change is caused in node $C_2$
Inference: Influence from concept $C_1$ to $C_2$ is positively high

Fig 15 shows the membership functions of the relationship between concepts $C_1$ and $C_2$, described by each one of the five experts respectively.

These linguistic variables (very high, very high, high, medium, high) are aggregated with the MAX method and the total linguistic weight is produced, which is transformed into a crisp value $w_{12}=0.6337$ (“C” point) after the CoA defuzzification method (Fig. 16).

Fig. 16 Aggregation of the three linguistic variables using the MAX method.

5. Conclusions
Input-Sensitive Fuzzy Cognitive Maps (IS-FCMs) and their ability to take into consideration the initial conditions of the PV-System are presented in this paper. The need for the development of the appropriate rule-base is revealed. Furthermore, an interesting aspect of the interactive attribute of the photovoltaic system is introduced. The hyperbolic tangent is used as a threshold function. The drawback of the convergence restriction, which conventional FCMs had, is eliminated. The results are quite promising and they confirm the reliability and effectiveness of IS-FCMs in energy issues.

References


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