Estimation of Time Difference of Arrival (TDOA) for the Source Radiates BPSK Signal

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Abstract

Time difference of arrival (TDOA) technology has been widely used in positioning and navigation system recently. The position estimation of a source through determining time difference of arrival (TDOA) of its signal among distributed sensors has many applications in civil as well as in the military. According to civil aspect, in mobile communication is widely used TDOA to perform location of cell phones of their subscribers and mobile stations using fixed base stations and also in costal stations is used TDOA to estimate boats and ship's location using acoustic waves. According the military aspect, it used to locate enemy's emitting devices such as radars, communication devices in the battle field. In this research paper we analyse the performance of TDOA estimation for Binary phase shift keying (BPSK) signals, the whole scenario for estimating time-difference-of-arrival (TDOA) measurements was considered. The cross-correlation among arbitrary sensors is used to estimate TDOA also by exploiting the spectral characteristic of the received signals by considering the maximum likelihood generalized cross correlation (ML-GCC) the source will as unknown position emitting BPSK signal corrupted by the white Gaussian noise, The problem studied is time-difference of arrival estimation in a multipath channel. The TDOA measurement can used for solving the localization problem typically implies cross-correlating the noisy signals received at pairs of sensors deployed within reception range of the source. Correlation-based localization is severely degraded by the presence of multipath. In Simulation results show us that the proposed method for TDOA can achieve the Cramer-Rao lower bound (CRLB) accuracy compared with changing the signal-to-noise-ratio (SNR) the observation time and bandwidth (BW) of the signal and also show the good performance of accuracy of the proposed source localization methods.

Keywords: CC, ML-GCC, CRLB, time difference of arrival (TDOA), Wireless sensor network (WSN).

1. Introduction

A continuous research and development of source localization has led to achieving more precise and accurate solution to find the true position of an emitting device in the various applications for civil and military fields.

According to civil aspects, the Sensor networks are becoming increasingly popular for applications such as determining the position of the source of wireless transmission, determining the location of an acoustic source by using microphone array. We determining position of a mobile receiver with pre-knowledge of transmitting time and global positioning system (GPS) usually provide worldwide high accuracy position measurement. It requires to line of sight multiple satellites. Time difference of arrival (TDOA) measurements, as they are called, is also used in locating cell phones.

According Military aspects, the electronic warfare where the problem is to accurately locate enemy transmitters to be able to make appropriate countermeasures, the TDOA approach may here offer higher accuracy than classical localization approaches, where the TDOA measurement have larger accuracy than triangulation measurement.
Source localization can be based on time of arrivals (TOA), time difference of arrivals (TDOA) or angle of arrivals (AOA) or combination of them. When the source and receiver are moving or one of them is moving so the frequency difference of arrivals (FDOA) can be exploited to improve location accuracy. In the next paragraph, some techniques will be introduced; they are used to achieve solutions for problem source localization.

To estimate the Direction of Arrivals (DOA) algorithms are used that exploit the phase differences between closely spaced antenna elements of an antenna array and employ phase-alignment methods for beam/null string [2-4]. The spacing of antenna elements within the antenna array is typically less than half wavelengths of all received signals. This is required to produce phase differences on the order of \( \pi \) radians or less to avoid ambiguities in the DOA estimate. The resolution of DOA estimates improves as baseline distances between antenna elements increase. However, this improvement is at the expense of ambiguities. As a result, DOA estimation methods are often used with short baselines to reduce or eliminate the ambiguities and other times with long baselines to improve resolution. Although, the DOA methods offer practical solutions for wireless position location, they have certain drawbacks. For example accurate DOA estimates, it is crucial that the signals coming from the source to the antenna arrays must be coming from the Line-Of-Sight (LOS) direction. Another factor is the considerable cost of installing antenna arrays. The position location system may need regular calibration since a minute change in the physical arrangement of the array because of storms may result in considerable position location error as the absolute angular position of the array is used as a reference to the angle of arrival (AOA) estimates. This is a problem that would be unique to position location as this will not affect the interference rejection capability of the array. Hence, if the arrays are to be used for position location they would either need extremely rugged installation or some other method of continuous calibration for accurate DOA estimates. Another problem with this method is the complexity of the DOA algorithms. Although, there are some exceptions such as ESPRIT and MUSIC based on Eigenvalue decomposition (EVD), these algorithms usually tend to be highly complex because of the need for measurement, storage and usage of array calibration data and their computationally intensive nature.

It may be possible for the sensors to indirectly determine the time that the signal takes from the source to the receiver on the forward or the reverse link. This may be done by measuring the time in which the source responds to an inquiry or an instruction transmitted to the mobile from the base station. The total time elapsed from the instant the command is transmitted to the instant the source response detected, composed of the sum of the round trip signal delay and any processing and response delay within the emitting unit. If the processing delay for the desired response within the emitter is known with sufficient accuracy, it can be subtracted from the total measured time, which would give us the total round trip delays. Half of that quantity would be an estimate of the signal delay in one direction, which would give us the approximate distance of the mobile from the sensor. If the emitter can be detected at two additional receivers then the position can be fixed by the triangulation method[5].

There are certain problems that this method could face. The estimate of the response delay within the source might be difficult to determine in practice. The main reason would be the variations in the designs of the handsets from different manufacturers. Secondly, this method is highly susceptible to timing errors in the absence of LOS, as there would be no way to reduce the errors induced because of multiple signal reflections on the forward or the reverse link.

The system that uses Time Difference of Arrivals (TDOA) to find a source location it requires at least three sensors one of them is a master (reference) and the other two are slave (Auxiliary) sensors [5, 6].

The principle of this system to measure the time difference of an intercept signal arriving slave sensor and the reference sensor for more details. TDOA systems basically solve the equation velocity times time equals distance, \( \nu t = d \), or more specifically, \( \nu \delta t = \delta \ell \), where \( \delta t = (t_i - t_f) \) is the difference between the arrival time \( t_i \) at sensor (i) and the source time \( t_f \), and \( \delta \ell \) is the distance between the measurement location \( (x_i, y_i) \) and source location \( (x, y) \). After that two curves (ns-1) curves, where (ns) the number of sensors) is obtained, the point of intersection of the curves is the location of emitter some previous work had used Hyperbolic location theory to evaluate curves, the hyperbola is the set of points at a constant range difference from two foci and each sensor pair gives a hyperbola on which the emitter lies then emitter location estimation is the intersection of all hyperbolas.

In addition the sensors in simplified case arranged linearly or are distributed arbitrary in complex cases. Large number of sensors achieves high accuracy and performance for location estimation with more complexity of calculations.

Most researchers work to develop, they modify algorithms that used to find an emitter position estimate that minimize its deviations from the true position. We are solving the problem the reliable and accurate position location of cell phones in mobile communication.
2. Related Work:

There are several different techniques to proposed source localization methods in time-difference-of-arrival (TDOA) measurements. Y.T. Chan et al. first purposes observe the use of two spatially separated receivers to resolve the presence of a distant signal source and its relative bearing\(^{[9]}\). Guosong Zhang et al. proposed passive-phase conjugation uses a channel probe signal transmitted prior to the data signal in order to estimate the channel response and multipath channel, there are paths that undergo incoherent scattering by the sea surface, and they decrease the coherence between the estimated channel response and the channel response for data signal\(^{[8]}\). Shuanglong Liu et al. purpose the analyses the basic principle of acoustic source localization using time difference of arrival. Time difference measurement is a key problem in TDOA location method for its accuracy directly determines the position estimation accuracy of the location system. Generalized cross-correlation (GCC) method can weaken the impact of noise on the delay estimation accuracy so this method is adopted in his paper to obtain a TDOA measurement by detecting the peak position of the correlation function of two signals. An improved Taylor algorithm with 10 iterations is proposed to solve the location coordinate of the acoustic source\(^{[9]}\). Enyang Xu et al. purpose he investigate robust and low complexity solutions to the problem of source localization based on the time-difference of arrivals (TDOA) measurement model. By adopting a min-max approximation to the maximum likelihood source location estimation and he develops two low complexity algorithms that can be reliably and rapidly solve through semi-definite relaxation.

3. Techniques and System Model:

The TDOA estimation methods will be explained in more details and discusses different algorithms that are used to estimate the time difference of arrivals and to solve the resulting hyperbolic equations. We compare those algorithms that can be used in communication and radar systems. We measure used to measure the accuracy of TDOA estimation and introduce the measure of accuracy used throughout in this paperwork.

3.1 Position Location based on TDOA Method

Hyperbolic position location (PL) estimation is accomplished in two stages. The first stage involves estimation of the TDOAs of the signal from a source, between pairs of receivers through the use of time delay estimation techniques conventional cross correlation CC and Generalized cross correlation GCC will be used to achieve high resolution TDOAs estimation when the white Gaussian noise is effected the source signal. In the second stage, the estimated TDOAs are transformed into noisy range difference measurements between sensors, resulting in a set of nonlinear hyperbolic equations. The second stage efficient algorithms to produce an unambiguous solution to these nonlinear equations. The solution provided by these equations results in the estimated position of the source. The following is a survey of different techniques that are used to estimate the TDOAs. After that is a similar survey of the techniques and algorithms that have been proposed to accurately solve the nonlinear hyperbolic equations in this paper focused only for the first part.

3.2 TDOA Estimation Techniques

The TDOA of a signal can be estimated by two general methods: subtracting time of arrivals TOA measurements from two base stations to produce a relative time difference of arrivals TDOA or through the use methods based on cross-correlation techniques, in which the received signal at one sensor (the reference) is correlated with the received signal at another sensor\(^{[10-13]}\). The first method applicable if the absolute TOA measurements are available, there doesn't seem to be any advantage in converting TOA measurements into TDOA measurements, as the position of the source could be triangulated using the TOA measurements directly. However, this may give us some increased accuracy when errors due to multiple signals reflect in pairs of TOA measurements are positively correlated because of having a common signal reflector. The more similar errors in pairs of TOAs if they are more it can be acquired by changing them into TDOAs. However, this is practical only when the TOA can be estimated by having knowledge of transmission time. If the timing reference is not available at the transmitter such as the Electronic warfare (EW) applications, then this method for estimating TDOAs cannot be used because of the absence of a timely reference on the source-to-be-located, the most commonly used technique for TDOA estimation is the cross-correlation based methods. The time requirement for this method is synchronization among all receivers participating in the TDOA measurements, which is more practical to achieve in most position location applications because of these factors we will discuss in details the cross-correlation technique for estimating TDOAs as well as the generalized cross-correlation methods.

3.3 Cross-Correlation (CC) Technique

Cross-Correlation is a measure of similarity of signal by another signal the autocorrelation is a special case of correlation when the measure of self-similarity of a signal with its delayed version considers. Cross correlation (CC) methods cross correlates prefiltered version of the received signals at two sensors through filters with the proper frequency response then correlated, integrated and squared. This is performed arrange a range of time shifts.
3.4 Generalized cross-correlation:

Conventional cross-correlation techniques that have been introduced to solve the problem of time difference of arrival (TDOA) estimation are referred to as Generalized Cross-Correlation (GCC) methods. These methods have been explored in [10,11]. These GCC methods cross-correlate pre-filtered versions of the received signals at two receiving stations, then estimate the TDOA \( \tau \) between the two stations as the location of the peak of the cross-correlation estimate. Pre-filtering is prepared to standout frequencies for which Signal-to-Noise Ratio (SNR) is low and attenuate the noise power before the signal is passed to the correlator.

\[
\hat{R}_{x_1x_2}(m) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) s(n+m) \quad (5)
\]

The cross-power spectrum is obtained by taking the Fourier transform of cross-correlation to improve the precision of TDOA estimation, the power spectrum of signal gives the distribution of the signal power among various frequencies and shows the existence, and also the relative power and random structure of signals, it is better to pre-filter received signals passing through the filter has response \( H_j(f) \) appropriates with the spectrum of the signals and narrow band, where \( j=1,2 \), because of the finite observation the estimated cross-power spectral only will be obtained and then apply an inverse Fourier transform to obtain estimated cross-correlation and finally estimated TDOA. As shown is the figure 2.1 each signal \( x_1(t) \) and \( x_2(t) \) are filtered through \( h_1 \) and \( h_2 \) then correlated integrated and squared. This is performed for a range of time shift \( t \) until a peak correlation is obtained. The time delay causing the cross-correlation peak is an
estimate of the TDOA. If the correlator is to provide an unbiased estimate of TDOA, the filters must exhibit the same phase characteristics and hence are usually taken to be identical filters \([2, 14, 15]\) when the two signals are filtered, the cross-power spectrum between the filtered outputs given by

\[ G_{\eta_1 \eta_2}(f) = H_1(f)H_2^*(f)G_{\eta_1 \eta_2}(f) \quad (6) \]

Where (*) denotes to complex conjugate, Therefore, the GCC given by the inverse Fourier transform of (6)

\[ R_{\eta_1 \eta_2}^G(\tau) = \frac{\Psi_G(f)G_{\eta_1 \eta_2}(f)e^{j2\pi f \tau}}{\int_{-\infty}^{\infty} \dh \psi_G(f)G_{\eta_1 \eta_2}(f)e^{j2\pi f \tau}} \quad (7) \]

Where \( \psi_G(f) = H_1(f)H_2^*(f) \) denotes the general frequency weighting, or filter function. Because only an estimate of \( \hat{G}_{\eta_1 \eta_2}(\tau) \) can be obtained from finite observation of \( x_1(t) \) and \( x_2(t) \) then

\[ \hat{R}_{\eta_1 \eta_2}^G(\tau) = \frac{\Psi_G(f)\hat{G}_{\eta_1 \eta_2}(f)e^{j2\pi f \tau}}{\int_{-\infty}^{\infty} \dh \psi_G(f)G_{\eta_1 \eta_2}(f)e^{j2\pi f \tau}} \quad (8) \]

The GCC method uses a weighting function \( \psi_G(f) \) to remove the effects of noise and interference, in case of using narrow band signal it achieves better TDOA estimation Using GCC; here in our work BPSK signal has been considered. The table shows the GCC frequency function.

We usually calculate the spectral quantities using the discrete Fourier transform (DTF) or the fast Fourier transform (FFT) as have been written in the above paragraphs.

**Table: The GCC frequency function**

<table>
<thead>
<tr>
<th>Processor Name</th>
<th>Frequency weighted function ( \psi_G(f) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-correlation</td>
<td>1</td>
</tr>
<tr>
<td>Roth Impulse response</td>
<td>( \sqrt{G_{\eta_1 \eta_2}(f)} ) or ( \sqrt{G_{\eta_1 \eta_2}(f)} )</td>
</tr>
<tr>
<td>Smoothed coherence Transform SCOT</td>
<td>( \sqrt{G_{\eta_1 \eta_2}(f)G_{\eta_1 \eta_2}(f)} )</td>
</tr>
<tr>
<td>Eckart</td>
<td>( G_{\eta_1 \eta_2}(f) ) |</td>
</tr>
<tr>
<td>Hannan Thomson or Maximum Likelihood</td>
<td>( \frac{\left</td>
</tr>
</tbody>
</table>

All these frequency functions have been derived in\([10]\) in our research the conventional CC and Maximum likelihood (ML) GCC will be considered. The CC has already explained it. In addition when the \( H_1(f)H_2^*(f) = 1 \), that means the weighed frequency function and that makes the conventional CC processor, other processors include the Roth impulse Response processor, the smoothed coherence transform (SCOT), the Eckart and Hannan-Thomonsen or Maximum likelihood processors (ML).

### 3.5 TDOA accuracy and Methods performance evaluation

The measurement of MRSE comparing with CRLB is a common method to evaluate performances of TDOA measurement, K. C. Ho. and Friedlander, Benjamin\([16, 17]\) have been introduced the formula of CRLB for BPSK signals and the CRLB for ML-GCC method has been driven in \([10]\) by using the spectral Characteristic of BPSK signal that affected with white Gaussian noise have used, then the probability density function of this model becomes Gaussian, by considering log likelihood function and doing calculations the for our model signal in equation (2)

\[ f(x(t), 0 < t < L, \theta) = k \cdot e^{-\frac{(1)}{2\sigma^2}(x(t) - s(t; \theta))^2} \quad (9) \]

\[ \ln f(x(t), 0 < t < L, \theta) = \ln k - \frac{1}{2\sigma^2} \int_0^L (x(t) - s(t; \theta))^2 \, dt \quad (10) \]

The CRLB is the inverse of the Fisher information matrix defined as

\[ J = -E \left[ \frac{\partial^2 \ln f}{\partial \hat{\theta} \partial \theta^*} \right] \quad (11) \]

The exact form of CRLB of BPSK signal in case of ML-GCC method introduced by \([10, 17]\)

\[ CRLB(\hat{\theta}) = \left( \int \frac{\left| \gamma_{12}(f) \right|^2}{1 - \left| \gamma_{12}(f) \right|^2} df \right)^{-1} \quad (12) \]

### 4. Experimental Results:

We describe the simulation that used to obtain the results of this work; we look at our signal model, in this work we assume the narrowband BPSK signal is radiated from the source in unknown position, this signal intercept by the sensors in arbitrary known positions, MATLAB is a powerful engineering software that used in a lot of scientific fields was used in this work, all figures are plotted and simulated by it.

To further explain the capabilities of the GCC method, in this chapter its resolution or precision performance is compared to the resolution performance of the conventional CC method in order of estimate TDOA. The major advantage of the GCC method over CC method is able to provide resistance to noise and interference and the ability to resolve multipath signal.
The designed narrowband, baseband BPSK signal that has been considered as a radiated signal from Unknown position far stationary target with normalized amplitude the parameter of the BPSK intercepted signals the following, the sampling frequency is 100 kHz, The actual time delay is $td = 40.16654578945465e^{-6}$ sec and the assumption for noise will be white Gaussian noise with zero mean; our assumption does not consider any relative motion among sensors or between source and sensors so we just consider the simplest scenario in our work.

4.1 The performance evaluation of TDOA estimation Methods

We apply the methods described in section three to our proposed BPSK generated signal corrupted by zero mean white Gaussian noise Our goal is to see how well we can estimate useful TDOAs measurements. The Root Mean Squared Error (RMSE) is used as the measure of improvement. We define the RMSE as the root square of the difference between the true TDOA value and the estimated TDOA value from the definition of RMSE with considering the CRLB that described in the section 3 in the equation (12). The square root of a margin between the true TDOA with the estimate TDOA. We compare the RMSE obtained from the ML-GCC, to RMSE for conventional CC method TDOA estimation.

To study the performance of TDOA estimator the affection of white Gaussian noise and the spectral characteristic of the signal will be considered. In simulation two ways have been used in simulation to calculate the power spectrum density PSD for BPSK signal and its noise component, the PSD does not concentrate in one frequency, due to the nature of a BPSK signal in the baseband, the PSD is defined as a Sinc function, the most power of signal concentrated in the major loop and the rest are small amount of power distributed in others side loops, so we ignore the infinite interval of the spectrum and focus only in the bandwidth from the $-B/2$ to $B/2$ that contain the most power of the signal where B denotes the bandwidth.

The RMSE versus signal to noise ratio SNR has been considered, the SNR arranged from -10dB to 15dB to show the difference between two methods in low and high SNR, because the effect of weighting function the ML-GCC curve close to CRLB level and CC not, with increasing SNR the conventional CC converge to the ML-GCC so we can observe high SNR no need for weighting, whatever it is better to use the ML-GCC rather than conventional CC to guarantee good performance when the SNR is low or high. For more proof considers the PSD of the signals.

\begin{align}
G_{x_1} (f) & = G_{x} (f) + G_{n_1} (f) \\
G_{x_2} (f) & = G_{x} (f) e^{-j2\pi f \tau} + G_{n_2} (f) \\
\end{align}

Assume that there is no correlation between signal and noise and the power spectrum of noises are equal under this assumption the magnitude square coherence from the definition in the equation

\begin{align}
\Psi_{HT} (f) = & \frac{G_{x} (f)}{(G_{x} (f) + G_{n} (f))^2} \approx \frac{1}{2G_{x} (f) G_{n} (f)} = \frac{1}{2G_{n} (f)} \quad (14)
\end{align}

In case of SNR is high so $G_{x} (f) > G_{n} (f)$ the weighting function will be

\begin{align}
\Psi_{HT} (f) \approx G_{x} (f) & < G_{n} (f) \quad (15)
\end{align}

In case of SNR is low $G_{n} (f) > G_{x} (f)$ the weighting function will be

\begin{align}
\Psi_{HT} (f) \approx G_{x} (f) & < G_{n} (f) \quad (16)
\end{align}

From the proof the significant results that in case when the SNR is low and the noise is Gaussian the weighting function very sensitive so we can use the PSD of BPSK signal to weighted $G_{x_1} (f)$ in the equation (7) and provide a better estimation rather than CC method.

In case of high SNR we can use both CC and ML-GCC methods to provide a good TDOA estimation as we showed in the figures.

In figure 2, we observed improved of GCC with increasing the SNR but CC didn’t affect. Generally the ML-GCC method performance is much better than CC that due to its weighted function has strongly relative to the nature of noise and signals. Also in figure 3 shows the improvement of RMSE with varying bandwidth we set SNR equal 0dB, the value of Bandwidth has been set between 50kHz to 200 kHz, the significant result that with constant SNR the estimation of TDOA is becoming better with increasing bandwidth and ML-GCC still better that CC in all cases, also we can see the improvement of performance when the time of observation is increased, figure 4 shows the result by choosing SNR equal 0dB the RMSE of all methods are decreased while keeping advantage of GCC over CC method.

5. Conclusion

We presented some background information about TDOA estimation and the techniques that used to estimate TDOA measurement by subtracting TOA measurement or using correlation based methods, also we presented its applications in the real world include the civil aspects and military aspects after that; we presented different ways that used to solve the localization problems include DOA and AOA and their advantages and limitations. We presented
the idea of source localization using TDOA measurement, firstly we mentioned the TDOA estimation methods based on cross-correlation and, we mentioned the conventional cross-correlation CC and Generalized cross-correlation GCC specifically the ML-GCC that used to extract and estimate TDOA measurement from the signals that radiated from far unknown position emitters, we gave the mathematical proof of both methods and we saw the ML-GCC is strongly relatives with the spectral component of the signals, after that the position location estimation methods were presented in details the Taylor-series and Chan’s WLS methods.

References

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