# Fault Tolerant Control Design For Takagi-Sugeno Nonlinear Systems

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#### Abstract

In this paper, a new actuator fault tolerant control strategy is proposed for nonlinear Takagi-Sugeno systems. The proposed control law uses the estimated fault and the error between the faulty system state and a reference system state. A proportional integral observer with unknown inputs is conceived in order to estimate simultaneously states and actuator faults. The problem of design of observer and the fault tolerant control law is formulated using linear matrix inequalities. Simulation example is given to illustrate the proposed method.

**Keywords:** Takagi-Sugeno systems, fault estimation, proportional integral observer with unknown inputs, actuator fault, active fault tolerant control.

### **1. Introduction**

In most cases, processes are affected by faults that can have harmful effects on the normal behavior of the system. Their estimation is necessary in order to be used to develop a fault tolerant control law able to minimize their effects on the concerned system.

The problem of fault tolerant control strategy has been treated these last years. The existing strategies are distinguished into two classes: passive and active. The first one is called also robust control. In this technique, faults are considered as uncertainties [1], [3], [6]–[8].

The active fault tolerant control approach consists on adapting the control law using the information issued from the FDI block [1]. The active fault tolerant control has been studied essentially for linear systems [16] and descriptor linear systems [15]. However, it turns out that linear models describe the dynamic behavior of the system around an operating point. Hence, the use of nonlinear models becomes unavoidable because it allows an accurate representation of the system on a wide operating range [4]. Multiple model approach constitutes a tool largely used to model the nonlinear systems [2]. The Takagi-Sugeno representation is the most used structure in the multiple model approach. This technique provides a mean to generalize the developed tools for linear systems to nonlinear systems thanks to the convex sum property of the weighting functions. One can cite some recently researches in the fault tolerant control field for nonlinear systems [5], [14].

In this paper, a new approach of the active fault tolerant control of Takagi-Sugeno systems is proposed. Proportional integral observer with unknown inputs is used to estimate actuator faults assumed to be considered as unknown inputs. These unknown inputs are estimated simultaneously with the states of the system.

The paper is organized as follows. Section 2 presents a short introduction to Takagi-Sugeno systems. Section 3 is devoted to the design of a proportional integral observer with unknown inputs for continuous time Takagi-Sugeno systems. The new strategy of the fault tolerant control is proposed in section 4. Numerical example is given in order to show the performance of the proposed approach.

### 2. Takagi-Sugeno structure for modeling

Consider the following general form of nonlinear systems

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t)) \end{cases}$$
(1)

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  and  $y(t) \in \mathbb{R}^p$ . f and g are nonlinear functions.

The Takagi-Sugeno approach allows to represent the behavior of the nonlinear system (1) by the interpolation of a set of linear submodels. The Takagi-Sugeno structure is given by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = C x(t) \end{cases}$$
(2)



 $A_i \in \mathbb{R}^{n^{*n}}$  is the state matrix,  $B_i \in \mathbb{R}^{n^{*m}}$  is the matrix of input and  $C \in \mathbb{R}^{p^{*n}}$  is the output matrix of the system. r is the number of local models. The weighting functions  $\mu_i(\xi(t))$ are nonlinear and depend on the decision variable  $\xi(t)$ . These functions verify the convex sum property:

$$\sum_{i=1}^{\prime} \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \le \mu_i(\xi(t)) \le 1$$
(3)

Thus, it is possible to generalize the tools developed for linear systems to nonlinear systems thanks to properties (3).

# **3.** Design of proportional integral observer with unknown inputs

#### 3.1 Problem statement

In this section, a proportional integral observer with unknown inputs is synthesized for estimating the actuator faults. Consider the following nonlinear Takagi-Sugeno model affected by unknown inputs:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i d(t)) \\ y(t) = C x(t) \end{cases}$$
(4)

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  and  $y(t) \in \mathbb{R}^p$  are respectively the state vector, the input vector and the measured output.  $d(t) \in \mathbb{R}^q$  is the vector of unknown inputs.  $A_i$ ,  $B_i$ ,  $E_i$ , and C are known constant matrices with appropriate dimensions.

The proposed proportional integral observer with unknown inputs takes the following form:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i z(t) + G_i u(t) + L_i y(t) + H_i \hat{d}(t)) \\ \hat{x}(t) = z(t) + M y(t) \\ \dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\phi_i(y(t) - \hat{y}(t)) \end{cases}$$
(5)

where  $\hat{x}$  is the estimated state vector, z represents the state vector of the observer,  $\hat{y}$  is the estimated output vector and  $\hat{d}$  are the unknown inputs estimated.  $N_i$ ,  $G_i$ ,  $L_i$ ,  $H_i$ ,  $\phi_i$ , and M are defined so that the reconstructed state converges asymptotically to the actual state x(t).

One notes  $e(t) = x(t) - \hat{x}(t)$  and  $f(t) = d(t) - \hat{d}(t)$ .

The dynamic of the state estimation error is given by the following equality:

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i e(t) + (PA_i - N_i - K_i C)x(t) + H_i f(t)) + \sum_{i=1}^{r} \mu_i(\xi(t))((PB_i - G_i)u(t) + (PE_i - H_i)d(t))$$
(6)

with  $K_i = L_i - N_i M$ .

If the following conditions are fulfilled

$$N_i = PA_i - K_iC \tag{7a}$$

$$H_i = PE_i \tag{7b}$$

$$G_i = PB_i \tag{7c}$$

$$L_i = K_i + N_i M \tag{7d}$$

$$P = I - MC \tag{7e}$$

The state estimation error (6) is reduced to

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i e(t) + H_i f(t))$$
(8)

It is assumed that the faults are bounded and slowly varying, i.e.  $\dot{d}(t) \approx 0$ .

The fault estimation error dynamics is given by:

$$\dot{f}(t) = -\sum_{i=1}^{r} \mu_i(\xi(t))\phi_i C(x(t) - \hat{x}(t))$$
(9)

The dynamics of the unknown inputs estimation error is given by:

$$\dot{f}(t) = -\sum_{i=1}^{r} \mu_i(\xi(t))\phi_i C e(t)$$
(10)

Equations (8) and (10) can be written in the following form:

$$\begin{bmatrix} \dot{e}(t) \\ \dot{f}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t)) \begin{bmatrix} N_i & H_i \\ -\phi_i C & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ f(t) \end{bmatrix}$$
(11)

The estimation error (11) converges asymptotically towards zero if the matrices  $\begin{bmatrix} N_i & H_i \\ -\phi_i C & 0 \end{bmatrix}$  are stable.

## 3.2 Method of resolution

Four steps are needed to determine the matrices of the multiple observer (5).

1. Knowing that 
$$P = I - MC$$
, we have:

$$I = P + MC = \begin{bmatrix} P & M \end{bmatrix} \begin{bmatrix} I_1 \\ C \end{bmatrix}$$
(12)

Matrix  $I_1$  is an identity matrix with appropriate dimension. Then, we obtain:

$$\begin{bmatrix} \mathbf{P} & M \end{bmatrix} = \begin{bmatrix} I_1 \\ C \end{bmatrix}^+ \tag{13}$$

Where 
$$\begin{bmatrix} I_1 \\ C \end{bmatrix}^+$$
 is the pseudo-inverse of  $\begin{bmatrix} I_1 \\ C \end{bmatrix}$ .

2. By determining P, one deduces:

$$G_i = PB_i$$
$$H_i = PE_i$$

3. To calculate the gains  $K_i$  and  $\phi_i$ , the estimation errors are rewritten as follows:

$$\begin{bmatrix} \dot{e}(t)\\ \dot{f}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t))(\bar{A}_i - \bar{K}_i\bar{C}) \begin{bmatrix} e(t)\\ f(t) \end{bmatrix}$$
(14)

Equation (14) is written as:

$$\dot{e}_{a}(t) = \sum_{i=1}^{r} \mu_{i}(\xi(t))(\bar{A}_{i} - \bar{K}_{i}\bar{C})e_{a}(t)$$
(15)

with

$$\overline{A}_i = \begin{pmatrix} PA_i & H_i \\ 0 & 0 \end{pmatrix}, \ \overline{K}_i = \begin{pmatrix} K_i \\ \phi_i \end{pmatrix}, \ e_a(t) = \begin{pmatrix} e(t) \\ f(t) \end{pmatrix},$$
$$\overline{C} = \begin{pmatrix} C & 0 \end{pmatrix}.$$

*Theorem 1:* The proportional integral observer with unknown inputs is determined if there exists a symmetric positive definite matrix X and matrices  $W_i = X\overline{K}_i$  such that the following LMI hold  $\forall i \in \{1, ..., r\}$ 

$$\overline{A}_i^T X + X \overline{A}_i - \overline{C}^T W_i^T - W_i \overline{C} < 0$$
<sup>(16)</sup>

The gains  $\overline{K}_i$  of the observer are computed from:

 $\overline{K}_i = X^{-1} W_i.$ 

4. The matrices  $N_i$  and  $L_i$  are deduced respectively from (7a) and (7d).

# 4. Fault tolerant control of Takagi-Sugeno systems

4.1 Problem formulation

The objective of this part is to conceive an actuator fault tolerant for nonlinear systems represented by Takagi-Sugeno models.

Let us consider the Takagi-Sugeno reference model without faults given by (2). The faulty system is described by:

$$\begin{cases} \dot{x}_{f}(t) = \sum_{i=1}^{r} \mu_{i}(\xi(t))(A_{i}x_{f}(t) + B_{i}u_{f}(t) + E_{i}d(t)) \\ y_{f}(t) = Cx_{f}(t) \end{cases}$$
(17)  
$$x_{f}(t) \in \mathbb{R}^{n} \text{ is the faulty state vector, } u_{f}(t) \in \mathbb{R}^{m} \text{ is the faulty output vector.} \\ \text{The aim is to design the control law } u_{f}(t) \text{ such that the system state } x_{f} \text{ converges towards the reference state } x \text{ .} \\ \text{Let us consider the proportional integral observer with unknown inputs that estimates simultaneously the state} \end{cases}$$

$$\begin{cases} \dot{z}_{f}(t) = \sum_{i=1}^{r} \mu_{i}(\xi(t))(N_{i}z_{f}(t) + G_{i}u_{f}(t) + L_{i}y_{f}(t) + H_{i}\hat{d}(t)) \\ \hat{x}_{f}(t) = z_{f}(t) + My_{f}(t) \\ \dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_{i}(\xi(t))\phi_{i}(y_{f}(t) - \hat{y}_{f}(t)) \end{cases}$$
(18)

and the faults of the system:

One proposes the following structure for the control law

$$u_f(t) = \sum_{i=1}^r \mu_i(\xi(t))(-\hat{Sd}(t) + w_i(x(t) - \hat{x}_f(t)) + u(t))$$
(19)

The matrices  $w_i$  are determined in order to minimize the state error between  $x_f$  and x. By analyzing the law  $u_f$ , the estimation of the faulty state vector  $x_f(t)$  and faults d(t) are needed.

Let us define  $\tilde{x}(t)$  the error between the states x(t) and  $x_f(t)$ ,  $\tilde{x}_f(t)$  the estimation error of the state  $x_f(t)$  and  $\tilde{d}(t)$  the fault estimation error

$$\tilde{x}(t) = x(t) - x_f(t)$$

$$\tilde{x}_f(t) = x_f(t) - \hat{x}_f(t)$$

$$\tilde{d}(t) = d(t) - \hat{d}(t)$$
The dynamics of  $\tilde{x}(t)$  is given by:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(t)) \mu_j(\xi(t)) ((A_i - B_i w_j) \tilde{x}(t) + B_i S \hat{d}(t) - E_i d(t) - B_i w_j \tilde{x}_f(t))$$
(20)

Choosing S such that  $E_i = B_i S$ , the dynamics of  $\tilde{x}(t)$  becomes

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(t)) \mu_j(\xi(t)) ((A_i - B_i w_j) \tilde{x}(t) - E_i \tilde{d}(t) - B_i w_j \tilde{x}_j(t))$$
(21)

The dynamics of  $\tilde{x}_f(t)$  is governed by the following differential equation:

$$\dot{\tilde{x}}_{f}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i}(\xi(t)) \mu_{j}(\xi(t)) (N_{i}\tilde{x}_{f}(t) + H_{i}\tilde{d}(t))$$
(22)  
with

$$P = I - MC, K_i = L_i - N_iM, G_i = PB_i, H_i = PE_i$$
  
and  $N_i = PA_i - K_iC$ .

The dynamics of  $\tilde{d}(t)$  is given by:

$$\dot{\tilde{d}}(t) = \dot{d}(t) - \sum_{i=1}^{r} \mu_i(\xi(t)) \phi_i C \tilde{x}_f(t)$$
(23)

A new augmented system can be written from (21), (22) and (23):

$$\dot{\psi}(t) = A_m \psi(t) + B_m \varphi(t) \tag{24}$$

where

$$\psi(t) = \begin{pmatrix} \tilde{x}(t) \\ \tilde{x}_f(t) \\ \tilde{d}(t) \end{pmatrix}, \ \varphi(t) = \dot{d}(t)$$
(25)

$$A_{m} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i}(\xi(t))\mu_{j}(\xi(t))A_{mi} \text{ and}$$
$$B_{m} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i}(\xi(t))\mu_{j}(\xi(t))B_{mi}$$

with

$$A_{mi} = \begin{pmatrix} A_i - B_i w_j & -B_i w_j & -E_i \\ 0 & N_i & H_i \\ 0 & -\phi_i C & 0 \end{pmatrix} \text{ and } B_m = \begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix}.$$

Gains  $\tilde{K}_i = \begin{pmatrix} K_i \\ \phi_i \end{pmatrix}$  and matrices  $w_j$  are determined by

solving the LMIs given by the following theorem (2).

Theorem 2: The system (24) describing the evolution of the errors  $\tilde{x}(t)$ ,  $\tilde{x}_f(t)$  and  $\tilde{d}(t)$  is stable if there exist symmetric and positive definite matrices Q,  $P_2$ , matrices  $F_j$  and  $g_i$  so that the following LMIs are verified, for  $i, j \in \{1, ..., r\}$ .

$$\begin{bmatrix} QA_i^T + A_iQ - B_iF_j - F_j^TB_i^T & \tilde{E}_{ij} \\ \tilde{E}_{ij}^T & \tilde{A}_i^TP_2 + P_2\tilde{A}_i - g_i\tilde{C} - \tilde{C}^Tg_i^T \end{bmatrix} < 0$$
(26)

The gains of the observer are computed from  $\tilde{K}_i = P_2^{-1}g_i$ and the matrices  $w_j$  are obtained by  $w_j = F_j Q^{-1}$ .

#### Proof

The gains of the observer (18) and the matrices  $w_i$  of the control law (19) are obtained using the Lyapunov theory. Let us choose the following Lyapunov function:

$$V(t) = \psi^{T}(t)P\psi(t), P = P^{T} > 0$$
<sup>(27)</sup>

*P* is a symmetric positive definite matrix that has a block diagonal form  $P = diag(P_1, P_2)$ .

The errors converge to zero if:

$$\dot{V}(t) < 0$$
, i.e.:  $A_{mi}^T P + P A_{mi} < 0$ 

The matrices  $A_{mi}$  and  $B_{mi}$  can be rewritten as

$$A_{mi} = \begin{pmatrix} A_i - B_i w_j & \tilde{E}_{ij} \\ 0 & \tilde{A}_i - \tilde{K}_i \tilde{C} \end{pmatrix}, \quad B_{mi} = \begin{pmatrix} 0 \\ \tilde{I} \end{pmatrix}$$
(28)

with:

$$\tilde{A}_{i} = \begin{pmatrix} PA_{i} & H_{i} \\ 0 & 0 \end{pmatrix}, \quad \tilde{K}_{i} = \begin{pmatrix} K_{i} \\ \phi_{i} \end{pmatrix}, \quad \tilde{C} = \begin{pmatrix} C & 0 \end{pmatrix},$$
$$\tilde{E}_{ij} = \begin{pmatrix} -B_{i}w_{j} & -E_{i} \end{pmatrix} \text{ and } \quad \tilde{I} = \begin{pmatrix} 0 \\ I \end{pmatrix}.$$

Let us introduce the following variables:

$$\Lambda_{ij} = (A_i - B_i w_j)$$
 and  $\Gamma_i = (\tilde{A}_i - \tilde{K}_i \tilde{C})$ .

 $\psi$  converges toward zero if there exist matrices  $P_1$  and  $P_2$  such that the following inequalities are satisfied

$$M_{ij} = \begin{bmatrix} \Lambda_{ij}^{\ T} P_1 + P_1 \Lambda_{ij} & P_1 \tilde{E}_{ij} \\ \tilde{E}_{ij}^{\ T} P_1 & \Gamma_i^{\ T} P_2 + P_2 \Gamma_i \end{bmatrix} < 0$$
(29)

Applying the congruence lemma as follows:

$$M_{ij} < 0 \Leftrightarrow \begin{pmatrix} P_1^{-1} & 0 \\ 0 & I \end{pmatrix} M_{ij} \begin{pmatrix} P_1^{-1} & 0 \\ 0 & I \end{pmatrix} < 0$$
(30)

The inequalities (29) become

$$\begin{bmatrix} P_{1}^{-1}\Lambda_{ij}^{T} + \Lambda_{ij}P_{1}^{-1} & \tilde{E}_{ij} \\ \tilde{E}_{ij}^{T} & \Gamma_{i}^{T}P_{2} + P_{2}\Gamma_{i} \end{bmatrix} < 0$$
(31)

Substituting  $Q = P_1^{-1}$ , one obtains:

$$\begin{bmatrix} Q\Lambda_{ij}^{T} + \Lambda_{ij}Q & \tilde{E}_{ij} \\ \tilde{E}_{ij}^{T} & \Gamma_{i}^{T}P_{2} + P_{2}\Gamma_{i} \end{bmatrix} < 0$$
(32)

With the changes of variables  $F_j = w_j Q$  and  $g_i = P_2 \tilde{K}_i$ , (32) becomes:

$$\begin{bmatrix} \lambda_{ij} & \tilde{E}_{ij} \\ \tilde{E}_{ij}^T & \gamma_i \end{bmatrix} < 0$$
(33)

where

$$\lambda_{ij} = QA_i^T + A_iQ - B_iF_j - F_j^TB_i^T$$
  

$$\gamma_i = \tilde{A}_i^TP_2 + P_2\tilde{A}_i - g_i\tilde{C} - \tilde{C}^Tg_i^T$$
  
Then, matrices  $w_i$  and  $\tilde{K}_i$  are derived from

$$w_j = F_j Q^{-1} \text{ and } \tilde{K}_i = P_2^{-1} g_i$$
 (34)

### 4.2 Simulation example

To illustrate the proposed actuator fault tolerant control strategy for Takagi-Sugeno systems, let us consider the multiple model (17) made up of two local models and involving three states and three outputs with  $\xi(t) = u(t)$ , where

$$A_{1} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, A_{2} = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$B_{1} = B_{2} = \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ -1 & 0 \end{bmatrix}, E_{1} = E_{2} = \begin{bmatrix} 0 & 3 \\ -1 & 1 \\ 3 & 0 \end{bmatrix},$$

The known input u(t) is defined by  $u(t) = [u_1^T(t) \quad u_2^T(t)]^T$ , with:

$$u_{1}(t) = \begin{cases} 0.6 & for \ t \le 20s \\ 0 & for \ 20s < t \le 30s \\ 0.6 & for \ 30s < t \le 60s \\ 0 & for \ 60s < t \le 80s \\ 0.6 & for \ t > 80s \end{cases}$$
  
and  $u_{2}(t) = \begin{cases} 0 & for \ t \le 20s \\ 0.4 * \sin(\pi t) & for \ 20s < t \le 55s \\ 0 & for \ 55s < t \le 75s \\ 0.3 & for \ t > 75s \end{cases}$ 

The weighting functions depend on the input u(t). They have been created on the basis of Gaussian membership

 $\begin{array}{c} 1 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 50 \\ 100 \\ \end{array}$ 

functions. Figure (1) shows their time evolution.

Fig. 1 Weighting functions

The gains of the observer and the matrices  $w_j$  that satisfy the conditions expressed in the theorem (2) are given by:

$$K_{1} = \begin{bmatrix} 16.96 & 5.78 & -36.70 \\ 20.06 & -44.04 & -5.93 \\ 10.52 & -2.48 & 9.70 \end{bmatrix},$$
  

$$\phi_{1} = \begin{bmatrix} -20.31 & 354.77 & -36.79 \\ 74.29 & 273.47 & -351.57 \end{bmatrix},$$
  

$$K_{2} = \begin{bmatrix} 28.01 & -8.24 & -45.48 \\ 21.48 & -53.72 & -2.92 \\ 14.31 & 1.93 & 4.13 \end{bmatrix},$$
  

$$\phi_{2} = \begin{bmatrix} 40.12 & 425.54 & -166.16 \\ 174.68 & 210.12 & -453.55 \end{bmatrix},$$
  

$$w_{1} = \begin{bmatrix} -44.79 & 69.93 & 46.65 \\ -55.74 & 55.14 & 3.31 \end{bmatrix} \text{ and }$$
  

$$w_{2} = \begin{bmatrix} -44.79 & 69.93 & 46.65 \\ -55.74 & 55.14 & 3.31 \end{bmatrix}.$$

Simulation results are shown in figures (2) to (5).

The proposed observer provides the state and fault estimation which errors are shown in the figure (3) and fault estimation in the figure (4). The state errors between the state of the system and those of the reference model are depicted in the figure (2). From this figure, one notes that



the system trajectory follows the reference trajectory even in the presence of actuator fault.

In order to conceive the proposed fault control strategy, the estimation of state and fault is required. This control law is given by the figure (5).

Thus, the proposed fault tolerant control law compensates the actuator fault and allows normal operation of the system even if a fault occurs.

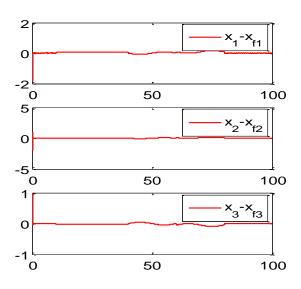
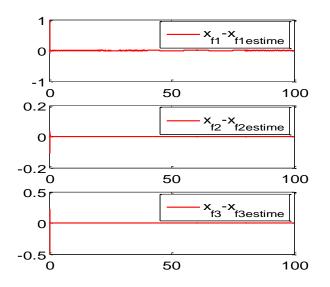
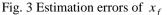
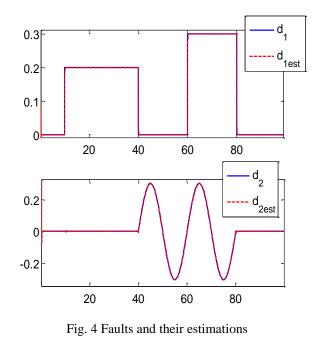


Fig. 2 Errors between x and  $x_f$ 







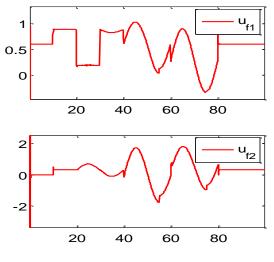


Fig. 5 Fault tolerant control  $u_f$ 

# 5. Conclusion

This paper deals with the problem of actuators faults tolerant control for nonlinear Takagi-Sugeno systems. The proposed control law is designed in order to guarantee the convergence of the states of the faulty system to the states of the reference model. To estimate actuators faults, a proportional integral observer with unknown inputs is used. The stability is studied by Lyapunov theory and LMI constraints are provided to design the gains matrices. Future works will be interested to develop the fault tolerant control law in the case of actuators and sensors faults.

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