

Reliability Analysis of Modified Irregular Augmented Shuffle Exchange Network (MIASEN)

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Abstract

Multistage Interconnection Networks (MINs) play a vital role to accomplish high performance in the field of multiprocessing systems, parallel and distributed systems, networks-on-chips, broadband communications, and very large scale integration (VLSI) designs. A MIN is more reliable if it is able to handle the more faults encounter in different switching stages. In this paper, reliability of a MIN is investigated in terms of upper and lower bounds of Mean Time to Failure (MTTF) and a new network Modified Irregular Augmented Shuffle Exchange Network (MIASEN) has been proposed. The performance and comparison analysis shows that the proposed network is more reliable and fault tolerant than the existing Irregular Augmented Shuffle Exchange Network-2 (IASEN-2).

Keywords: Fault tolerance, reliability, multistage interconnection network.

1. Introduction

With advances in VLSI technology, a greater number of multiple-processor are used to accomplish high performance computation. There are two types of parallel computers interconnection networks: Static and Dynamic. The first one is a point-to-point connection network in which connections don't change during program execution while in case of later the connections are dynamically configured on demand of program. The dynamic interconnection networks are three types: Crossbar networks, bus networks, and Multistage Interconnection Networks. Multistage Interconnection Network (MIN) performs a vital role in high performance computing like supercomputers. MIN is used to create a connection among memory elements at one side and processing elements at other side connected by many stages of switching elements. The memory elements are used to store data required by the processing elements and processing elements are responsible for computational parallelism. MIN provides faster speed with low cost in a multiprocessor system as compared to single-processor system. These networks are used in both Single Instruction Multiple Data (SIMD) and Multiple Instruction Multiple Data (MIMD) computers. MINs can be two types: single path MINs and multi-path MINs. In single path MIN, there is one-to-one connection between each source and destination pair e.g. banyan network [1], baseline [8], butterfly [2], delta networks [5], binary n-cube network[6], omega network[7] and shuffle-exchange network. In a multi-path MIN there occurs one-to-many path connection between source and destination e.g. clos network [9], Parallel Benes [2], and Non-blocking extended generalized shuffle (EGS) network etc. Fault tolerance, and reliability are responsible for the performance of a MIN. single path MINs are less costly than multi-path MINs, but are less fault tolerant and reliable that is a major issue. Fault tolerance, reliability and permutation capability are the important issues and factors, which are able to measure the performance of a MIN.

A number of research works have been done to design new networks and to increase the fault-tolerance in MIN [3], [4,5,6,7]. Various routing schemes and permutation capability and other issues related to routing have also been broadly researched [8,9,10], but a little research work has been done to the computation of reliability of these networks. Reliability is measured in terms of optimistic (or upper) bound and pessimistic (or lower) bound of Mean Time to Failure (MTTF). The simple series-parallel probabilistic combinations used to calculate reliability. In this research paper, a new MIN named Modified Irregular Augmented Shuffle Exchange Network (MIASEN) is has been proposed. The reliability of proposed Modified Irregular Augmented Shuffle Exchange Network (MIASEN) is compared with existing Irregular Augmented Shuffle Exchange Network-2 (IASEN-2).

The next section describes of design and basic structure of existing network and proposed Modified Irregular Augmented Shuffle Exchange Network (MIASEN). Section 3 focuses on the Fault-tolerance, and reliability aspects of MIASEN are analyzed. Section 4 concentrates on the cost, cost effectiveness of MINs is analyzed. In Section 5, the result and conclusion has been presented.



2. Structure and Design Of Multistage **Interconnection Networks**

The dynamic MINs can be classified into three categories based on their topology: Regular, Irregular and Hybrid MINs.

In regular MIN, there equal number of switching elements (SEs) in each stage, but in irregular MIN, the number of switching elements (SEs) is not same in each stage. The hybrid MIN is the combination of regular and irregular MIN i.e. it consists the characteristics of both regular and irregular MIN. In this paper, we are focusing on irregular MINs. The existing Irregular Augmented Shuffle Exchange Network-2 (IASEN-2) and proposed Modified Irregular Augmented Shuffle Exchange Network (MIASEN) are discussed below.

2.1 Irregular Augmented Shuffle Exchange Network-2 (IASEN-2)

Irregular Augmented Shuffle Exchange Networks-2 (IASEN-2) [10] has N sources and N destinations with n=(log₂N) stages. Each source and destination is associated with the multiplexers (MUX) of size 2x1 and demultiplexers (DEMUX) of size 1x2 respectively.



Fig. 1 Irregular Augmented Shuffle Exchange Networks-2 (IASEN-2).

The first and last stages are linked with N/2 switching elements (SE) [18]. The first stage and last stage have SEs of size 2x3 and 3x2 respectively but SE of second stage and third stage has size 9x3 and 3x9 respectively. The second and third stage consist N/8 SEs. The SEs of each stage is associated with each other through alternative

links. The first and last stages are linked with N/2 switching elements (SE) [18].

2.2 Modified Irregular Augmented Shuffle Exchange Network (MIASEN)

The Modified Irregular Augmented Shuffle Exchange Network (MIASEN) is an N×N size irregular multistage interconnection networks with [(log₂N)-1] number of stages. First and last stages have N/2 switching elements (SE) and middle stage has (N/8) number of switching elements. MIASEN has N sources and N destinations, which are connected, with N multiplexers (MUX) and N demultiplexers (DEMUX) respectively. In the first stage, middle stage and last stage, size of each SE is 3×3 , 5×5 , and 2×2 respectively. The size of each multiplexer (MUX) and demultiplexer (DEMUX) in MIASEN is 2×1 and 1×2 respectively. In first stage, each switching element (SE) is attached with two multiplexer of size 2x1 and in last stage; two demultiplexers of size 1x2 are connected with each SE. The 16x16 network size MIASEN is mentioned in fig. 2.



(MIASEN).

A conjugate loop is formed when the switches are interconnected via the auxiliary links. The two switches, which form a loop, have their individual conjugate switches in an alternate loop for example, switches A&C and B&D are conjugate loops, and switches A&B and A&D are conjugate switches in upper half of first stage of MIASEN [16]. The MIASEN is a fault tolerant and reliable MIN, if any failure occurs in any switch in the network then there will be an alternate path to work properly. MIASEN can be on-line repair and maintainable





because if a loop is removed from any stage, MIASEN will work properly.

3. Reliability Analysis of MIASEN and IASEN-2

The reliability analysis can be basically considered as hardware reliability [12] and software reliability [11]. In this paper, the focus is given to the hardware reliability of the networks. There are three types of fault models implemented to measure reliability of MINs:

(1) stuck-at fault (2) link-fault (3) switch-fault

In Stuck-at fault, if any failure occurs in a crossbar switching element it'll remain in a particular state regardless of the control inputs given to it. This is affecting its capability to setup suitable connections.

In link-fault model, a failure affects a specific link of a switching element, leaving remaining part of the switch operational.

The switch fault model (or dead-fault model) is the worst case out of the three fault model. In this, if a switch fails then it'll become totally useless and non-operational [13].

These all fault model focus on the failure of switches and the failure effects on switches. The "full access" criterion and "switch fault" model is taken here to measure (MTTF) Mean Time To Failure of MIASENs. The full access means capacity to reach from any input to any output precisely in one pass even some switching components may be faulty (i.e. crossbar switches, MUX, DEMUX) but not the entire network [14] and this failure of components doesn't affects the reliability of others i.e. switch failure occurs independently. Reliability of IASEN-2, MIASEN networks is analyzed in terms of pessimistic (or lower) bound, optimistic (or upper) bound and Mean time to Failure (MTTF). Mean time to Failure (MTTF) is the estimated time elapsed before some source is separated from some destination [19]. Reliability equations of proposed MIASEN are derived in terms of MTTF lower bound and upper bounds and these bounds are calculated using simple series-parallel probabilistic combinations. To calculate reliability we need some assumptions for the analysis of the failure rates of the components, which are as follows:

- The failure rate of a segment can be derived from its gate count. For 2×2 crossbar switches, the failure rate is λ (where λ is about 10⁻⁶ per hour) [15].
- We assume that the failure rate of m×1 MUX and 1×m DEMUX is m $\lambda/4$ i.e. $\lambda_m = \lambda_d = m\lambda/4$. Failure of the 2×1 MUX and 1×2 DEMUX occurs individually with failure rates of $\lambda_m = \lambda/2$ and $\lambda_d = \lambda/2$ respectively.

- According to the adaptive routing scheme, the 2×2 switch in the last stage and its connected 1×2 DEMUX are considered in a series system. So we consider these three components as single segment (SE_{2d}). Based on the gate count we assign failure rate to this group $\lambda_{2d}=2\lambda$ (2 × 2=4, $1\times2=2$, $1\times2=2$, total=8, 8/4=2).
- Let failure rate of the 5 × 5 switch is λ_5 and 3 × 3 switch is λ_3 , then based on gate count, λ_5 = 6.25 λ and λ_3 = 2.25 λ and λ_{3m} = 3.25 λ .
- Let failure rate of the 5 × 5 switch is λ_5 and 3 × 3 switch is λ_3 , then based on gate count, λ_5 = 6.25 λ and λ_3 = 2.25 λ .

3.1 Optimistic or Upper Bound of MIASEN

Each source is connected to two 2×1 MUX and each SE in the first stage has a conjugate pair in MIASEN. To calculate the upper bound we assumed that the MIASEN is working on condition that one of the two multiplexers attached to a source is operational and both components in a conjugate pair (switch or loop) are not faulty [14]. Therefore, we can say that even if the half of the component of a network (or one sub-network) is faulty even then MIASEN is still working. The upper bound block diagram of MIASEN is shown in fig. 3.



The reliability equations of upper bound are given below:

$$\begin{aligned} f1 &= \left[1 - (1 - e^{-\lambda_m t})^2\right]^{-N/2} \\ f2 &= \left[1 - (1 - (e^{-\lambda_3 t})^2)^2\right]^{-N/4} \\ f3 &= \left[1 - (1 - e^{-\lambda_3 t})^2\right]^{-N/16} \\ f4 &= \left[1 - (1 - e^{-\lambda_2 t})^2\right]^{-N/4} \end{aligned}$$

$$\begin{split} \text{Where, } \lambda_m &= \lambda/2, \, \lambda_3 = 2.25\lambda, \, \lambda_5 = 6.25\lambda, \, \lambda_{2d} = 2\lambda \\ R_{MIASEN_UB} &= f1 \, \ast \, f2 \, \ast \, f3 \, \ast \, f4 \end{split}$$

 $MTTF_{MIASEN-UB} = \int_0^{\infty} R_{MIASEN_UB}(t) dt$



3.2 Pessimistic or Lower Bound of MIASEN

To calculate the pessimistic or lower bound of MIASEN, let us assume that the MIASEN is failed whenever more than one conjugate loop has a defective switch or multiple conjugate switch in the last stage fails. At the input side of MIASEN, routing algorithm doesn't consider the 2x1 MUX to be the integral part of the 3 x 3 switch. Therefore, if we group two MUX with each switch at input side and consider them a series system (SE_{3m}) then we can say that if at least one of the MUX associated to a selected switch is working, the switch can still be used for routing [19]. The failure rate of SE_{3m} is λ_{3m} =3.25 λ . The block diagram of pessimistic or lower bound of MIASEN is shown in Fig. 4.



Fig. 4 Lower Bound of MIASEN

The reliability equations of lower bound are given below:

$$\begin{array}{l} f1 = [1\text{-}(1\text{-}e^{-\lambda_{3m}t})^{2}]^{-N/4} \\ f2 = [1\text{-}(1\text{-}e^{-\lambda_{2}t})^{2}]^{-N/16} \\ f3 = [1\text{-}(1\text{-}e^{-\lambda_{2}dt})^{2}]^{-N/4} \\ R_{\text{MIASEN_LB}} = f1 * f2 * f3 \\ \text{Where, } \lambda_{3m} \text{=} 3.25 \ \lambda, \ \lambda_{5} \text{=} 6.25\lambda, \ \lambda_{2d} \text{=} 2\lambda \end{array}$$

 $MTTF = \int_0^\infty R_{MIASEN_{LB}}(t) dt$

3.3 Optimistic or Upper Bound of IASEN-2

Following the similar procedure as referred in the preceding segment (upper bound of MIASEN), the MTTF optimistic or upper bound of the IASEN-2 may be illustrated the use of the reliability block diagram shown in Fig. 5.



Therefore, if we consider that the IASEN-2 is working so long as one of the two MUX connected to a switch is working and so long as a conjugate loop or conjugate switch isn't always faulty. Then we can certificates many as one half of the components to fail and the IASEN-2 might also still be working [14]. We can assume that the failure rate of 9x3 switching element (SE_{9,3}) is $\lambda_{9,3}$, failure rate of 3x9 switching element (SE_{3,9}) is $\lambda_{3,9}$ and failure rate of 2x3 switching element (SE_{2,3}) is $\lambda_{2,3}$. According to the gate counts of crossbar switch the failure rate of SE_{9,3} is $\lambda_{9,3}$ =6.75 λ , the failure rate of SE_{3,9} is $\lambda_{3,9}$ =6.75 λ and failure rate of SE_{2,3} is 1.5 λ .

Reliability equations for upper bound MTTF

$$\begin{split} \mathbf{f1} &= \left[1 - (1 - e^{-\lambda_{m} t})^{2}\right]^{N/2} \\ \mathbf{f2} &= \left[1 - (1 - (e^{-\lambda_{2,3} t})^{2}\right]^{N/4} \\ \mathbf{f3} &= \left[1 - (1 - e^{-\lambda_{3,9} t})^{2}\right]^{N/16} \\ \mathbf{f4} &= \left[1 - (1 - e^{-\lambda_{3,9} t})^{2}\right]^{N/16} \\ \mathbf{f5} &= \left[1 - (1 - e^{-\lambda_{2} dt})^{2}\right]^{N/4} \end{split}$$

Where,

$$\lambda_{m} = \lambda/2, \lambda_{2,3} = 1.5\lambda, \lambda_{9,3} = 6.75\lambda, \lambda_{3,9} = 6.75\lambda, \lambda_{2d} = 2\lambda$$

 $R_{IASEN-2_UB} = f1 * f2 * f3 * f4 * f5$

 $MTTF_{IASEN-2_UB} = \int_0^\infty R_{IASEN-2_UB}(t) dt$

3.4 Pessimistic or Lower Bound of IASEN-2

Following the similar, procedure as we used before to calculate the MTTF lower bound of the MIASEN can be illustrated using the reliability block diagram shown in Fig.(6).



Reliability equations for lower bound MTTF

$$f1 = [1-(1-e^{-\lambda_{3m}t})^2]^{N/4}$$

$$f2 = [1-(1-e^{-\lambda_{9,3}t})^2]^{N/16}$$

$$f3 = [1-(1-e^{-\lambda_{3,9}t})^2]^{N/16}$$

$$f4 = [1-(1-e^{-\lambda_{2d}t})^2]^{N/4}$$

Where

$$\lambda_{2,0} = 6.75\lambda$$
, $\lambda_{0,2} = 6.75\lambda$, $\lambda_{2,d} = 2\lambda$, $\lambda_{2,m} = 2.5\lambda$

 $R_{\text{IASEN-2_LB}} = f1 * f2 * f3 * f4$

$$MTTF_{IASEN-2_LB} = \int_0^{\infty} R_{IASEN-2_LB}(t) dt$$



The Relative deviations in the MTTF upper and lower bounds of IASEN-2 and MIASEN, are illustrated in Fig. 7 and Fig. 8 respectively.



Fig. 7 MTTF Optimistic (Upper) bound comparison of MIASEN and IASEN-2



Fig. 8 MTTF Pessimistic (Lower) bound comparison of MIASEN and IASEN-2

The graph and table shows that the MTTF upper and lower bound of MIASEN are higher than the IASEN-2 for different network sizes (N) (from N>=16 and so on). The results of MTTF reliability equations are shown in table-1.

Table 1: MTTF Upper Bound and Lower Bound of IASEN-2 and MIASEN for different Network Size

Network Size	IASEN-2		MIASEN	
(logN)	Upper	Lower	Upper	Lower
	Bound	Bound	Bound	Bound
4	4.9966	4.9964	4.9977	4.9971
5	4.9931	4.9931	4.9954	4.9942
6	4.986	4.9859	4.9909	4.9885
7	4.9724	4.9719	4.9817	4.977
8	4.945	4.9443	4.9637	4.9544
9	4.8911	4.8898	4.9277	4.9095

4. Cost Analysis

The cost a network can be calculated that the cost of a switch is number of gates involved which is proportional to the number of crosspoints within that switch [17]. For example, the cost of a 2x2 switch is 4 units of hardware. For mx1 MUX and 1xm DEMUX let the cost is m unit of hardware. The cost of proposed MIASEN with network size 16x16 is given in the Table 2.

Table 2: Cost of MIASEN						
Type of Component	Size	Total No. of Switch /MUX or DEMUX	Cost			
switch	3x3	8	3x3x8=72			
Switch	5x5	2	5x5x2=50			
Switch	2x2	8	2x2x8=32			
MUX	1x2	16	1x2x16=32			
DEMUX	2x1	16	2x1x16=32			
Co	218					

The cost of existing IASEN-2 with network size 16x16 is given in Table 3.

Table 3: Cost of IASEN-2						
Type of	Size	Total No. of Switch	Cost			
Component		/MUX or DEMUX				
switch	2x3	8	2x3x8=48			
switch	9x3	2	9x3x2=54			
switch	3x9	2	3x9x2=54			
switch	3x2	8	3x2x8=48			
MUX	1x2	16	1x2x16=32			
DEMUX	2x1	16	2x1x16=32			
C	268					

Thus, the cost of IASEN-2 is more than the cost of MIASEN.

5. Conclusion

In this paper, the proposed Modified Irregular Augmented Shuffle exchange Network (MIASEN) has accomplished significant fault tolerance and good reliability with relatively low cost as compared to IASEN-2. The results and analysis shows that MTTF upper and lower bound of MIASEN is always higher than that of IASEN-2. Therefore, we can say that MIASEN is having better reliability and more fault-tolerant than IASEN-2 network.

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