

# Image Segmentation Based on a Finite Generalized New Symmetric Mixture Model with K – Means

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## Abstract

In this paper a novel image segmentation and retrieval method based on finite new symmetric mixture model with K-means clustering is developed. Here it is considered that pixel intensities in each image region follow a new symmetric distribution. The new symmetric distribution includes platy-kurtic and meso-kurtic distributions. This also includes Gaussian mixture model as a particular case. The number of components (image regions) is obtained through K-means algorithm. The model parameters are estimated by deriving the updated equations of the EM algorithm. The segmentation of the image is done by maximizing the component likelihood. The performance of the proposed algorithm is studied by computing the segmentation performance metrics like, PRI, VOI, and GCE. The ability of this method for image retrieval is demonstrated by computing the image quality metrics for five images namely HORSE, MAN, BIRD, BOAT and TOWER. The experimental results show that this method outperforms the existing model based image segmentation methods.

**Keywords:** Image segmentation, EM algorithm, New Symmetric Distribution. Image Quality Metrics

## 1. Introduction

Segmentation is the main consideration for image analysis and image retrieval. With segmentation it is possible to identify the regions of interest and objects which are highly useful. Image segmentation is defined as the process of dividing the image into different image regions such that each region is homogeneous. Image segmentation can be classified into two categories namely, parametric and non-parametric image

segmentation. A more comprehensive discussion on image segmentation is given by (S.K.Pal and N.R.Pal (1993), Jahne (1995), and Cheng et al (2001)). There does not exist a single algorithm that works for all applications.

Model based image segmentation is more efficient compared to the non-parametric methods of segmentation. Recently, much emphasis is given for image analysis through Finite Gaussian Mixture Model (Yamazaki et al. (1998), T.Lie et al.(1993), N.Nasios et al.(2006), Z.H.Zhang et al.(2003)). In Finite Gaussian Mixture Model each image region is characterized by a Gaussian distribution and the entire image is considered to be a mixture of these Gaussian components. Here it is assumed that the whole image is characterized by Gaussian mixture model in which the pixel intensities of each image region follow a Gaussian distribution. For gray level images the pixel intensity is the most suitable feature for segmenting the image (S.K.Pal and N.R.Pal, (1993)).

However, in finite Gaussian mixture model the pixel intensities of the image region are considered to be meso-kurtic and symmetric. But in some images the pixel intensities of the image region may not be distributed as meso – kurtic even though they are symmetric. To have a more close approximation to the pixel intensities of each image region it is needed to consider that the pixel intensities of each region follow a more general symmetric distribution. Srinivasa Rao, et al., (1997) have introduced a new symmetric distribution which is capable of portraying several platy – kurtic distributions. It also includes Gaussian

as a particular case for a specific value of the index parameter. Hence, in this chapter an image segmentation algorithm is developed and analyzed with the assumption that the whole image is characterized by a finite mixture of new symmetric distribution in which the pixel intensities of each image region follows a new symmetric distribution.

In mixture models one of the important factors is the number of components K (regions). Usually the number of components are assumed to be known as apriori. This will generally effect the segmentation results. If this number deviates from true value of K then the misclassification of pixels in the image is very high. To have a more accurate analysis of the number of regions in the whole image, the K value is identified through the K – Means algorithm (Rose H.Turi, (2001)) along with the histogram of the pixel intensities.

Using the Expectation Maximization (EM) algorithm the model parameters are estimated. The segmentation algorithm is developed through maximizing the component likelihood. The performance of the segmentation algorithm is evaluated by obtaining performance measures like PRI, GCE and VOI by applying them on five images HORSE, MAN, BIRD, BOAT and TOWER. The performance of this algorithm is compared with the image segmentation algorithm based on Finite Gaussian Mixture Model with K-Means. The efficiency of it in image retrievals is also studied through obtaining the image quality metrics like, average difference, maximum distance, image fidelity, mean square error, signal to noise ratio and image quality index and comparing it with earlier algorithms.

## 2. Finite Mixture Of New Symmetric Distribution

In low level image analysis the entire image is considered as a union of several image regions. In each image region the image data is quantized by pixel intensities. The pixel intensity  $z = f(x, y)$  for a given point ( pixel )  $(x, y)$  is a random variable, because of the fact that the brightness measured at a point in the image is influenced by various random factors like vision, lighting, moisture, environmental conditions etc,. To model the pixel intensities the image region it is assumed that the pixel intensities of the region follows a new symmetric distribution given by Srinivasa Rao et al., (1997).

The probability density function of the pixel intensity is

$$f(Z, \mu, \sigma^2, r) = \frac{\left(2r + \left(\frac{z-\mu}{\sigma}\right)^2\right)^r e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}}{\alpha(2r)^r (2\pi)^{\frac{1}{2}} + \sum_{j=1}^r \binom{r}{j} (2r)^{-j} 2^{\frac{j-1}{2}} \Gamma(j+\frac{1}{2}) \sigma},$$

$$-\infty < Z < \infty, -\infty < \mu < \infty, \sigma > 0$$
(1)

For different values of the parameters the various shapes of probability curves associated with new symmetric distribution are shown in Figure 1.

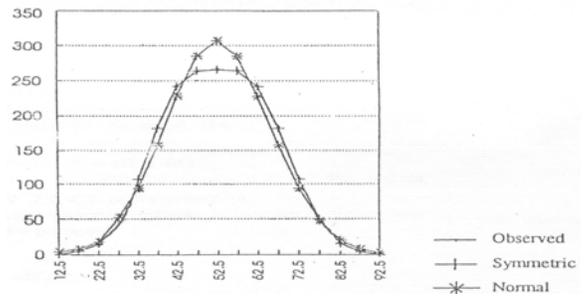


Figure1. Frequency curves of new symmetric distribution

Each value of the shape parameter 's' (= 0,1,2,3,...) gives a bell shaped distribution. For  $r = 0$  the equation reduces to a normal probability density function with parameter  $\mu$  and  $\sigma$ .

Its central moments are

$$\mu_{2n} = \frac{\left[ \Gamma(n+\frac{1}{2}) + \sum_{j=1}^r \binom{r}{j} r^{-j} \Gamma(n+j+\frac{1}{2}) \right]}{\left[ (\pi)^{\frac{1}{2}} + \sum_{j=1}^r \binom{r}{j} r^{-j} \Gamma(j+\frac{1}{2}) \right]} 2^n \sigma^{2n}$$
(2)

And  $\mu_{2n+1} = 0$

The kurtosis of the distribution is

$$\beta_2 = \frac{\left[ \frac{3}{4} \pi^{\frac{1}{2}} + \sum_{j=1}^r \binom{r}{j} r^{-j} (j+\frac{1}{2})(j+\frac{3}{2}) \Gamma(j+\frac{1}{2}) \right]}{\left[ \frac{\pi^{\frac{1}{2}} + \sum_{j=1}^r \binom{r}{j} r^{-j} \Gamma(j+\frac{1}{2})}{\left[ \frac{\pi^{\frac{1}{2}} + \sum_{j=1}^r \binom{r}{j} r^{-j} \Gamma(j+\frac{1}{2}) \right]^2} \right]} 2^2$$
(3)

The entire image is a collection of regions which are characterized by new symmetric distribution. Here, it is assumed that the pixel intensities of the whole image follows

a  $K$  – component mixture of new symmetric distribution and its probability density function is of the form

$$p(z) = \sum_{i=1}^K \alpha_i f_i(z / \mu_i, \sigma_i^2, r_i) \quad (4)$$

where,  $K$  is number of regions ,  $0 \leq \alpha_i \leq 1$  are weights such that  $\sum \alpha_i = 1$  and  $f_i(z, \mu, \sigma^2, r)$  is as given in equation ( 1 ).  $\alpha_i$  is the weight associated with  $i^{\text{th}}$  region in the whole image.

In general the pixel intensities in the image regions are statistically correlated and these correlations can be reduced by spatial sampling (Lie. T and Sewehand. W ( 1992 )) or spatial averaging ( Kelly P.A. et al.,( 1998 ) ). After reduction of correlation the pixels are considered to be uncorrelated and independent. The mean pixel intensity of the whole image is  $E(Z) = \sum_{i=1}^K \alpha_i \mu_i$ .

### 3. Estimation of the Model Parameter by EM Algorithm

In this section we derive the updated equations of the model parameters using Expectation Maximization (EM) algorithm. The likelihood function of the observations  $z_1, z_2, z_3, \dots, z_N$  drawn from an image is

$$L(\theta) = \sum_{s=1}^N \left( \sum_{i=1}^K \alpha_i f_i(z_s, \theta_i) \right),$$

$$L(\theta) = \sum_{s=1}^N \left[ \frac{\sum_{i=1}^K \alpha_i \left( 2r_i + \left( \frac{z_s - \mu_i}{\sigma_i} \right)^2 \right)^{r_i} e^{-\frac{1}{2} \left( \frac{z_s - \mu_i}{\sigma_i} \right)^2}}{\sigma_i (2r_i)^{r_i} (2\pi)^2 + \sum_{j=1}^{r_i} \binom{r_i}{j} (2r_i)^{r_i - j} 2^{j + \frac{1}{2}} \Gamma(j + \frac{1}{2}) \sigma_i} \right] \quad (5)$$

where,  $\theta = (\mu_i, \sigma_i^2, r_i, \alpha_i; i = 1, 2, \dots, K)$  is the set of parameters.

The expectation of the log likelihood function of the sample is

$$Q(\theta; \theta^{(l)}) = E_{\theta^{(l)}} [\log L(\theta) / \bar{z}]$$

This implies

$$Q(\theta; \theta^{(l)}) = \sum_{i=1}^K \sum_{s=1}^N (t_i(z_s, \theta^{(l)}) (\log f_i(z_s, \theta) + \log \alpha_i)) \quad (6)$$

The updated equation of  $\alpha_i$  at  $(l+1)^{\text{th}}$  iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N} \sum_{s=1}^N t_i(z_s, \theta^{(l)})$$

$$= \frac{1}{N} \sum_{s=1}^N \left[ \frac{\alpha_i^{(l)} f_i(z_s, \theta^{(l)})}{\sum_{i=1}^K \alpha_i^{(l)} f_i(z_s, \theta^{(l)})} \right] \quad (7)$$

The updated equation of  $\mu_i$  at  $(l+1)^{\text{th}}$  iteration is

$$\mu_i^{(l+1)} = \frac{\sum_{s=1}^N z_s t_i(z_s, \theta^{(l)}) - \sum_{s=1}^N t_i(z_s, \theta^{(l)}) \left( \frac{2r_i \sigma_i^{2(l)} (z_s - \mu_i^l)}{2r_i \sigma_i^{2(l)} + (z_s - \mu_i^l)^2} \right)}{\sum_{s=1}^N t_i(z_s, \theta^{(l)})} \quad (8)$$

where,

$$t_i(z_s, \theta^{(l)}) = \frac{\alpha_i^{(l+1)} f_i(z_s, \mu_i^{(l)}, (\sigma_i^2)^l, r_i)}{\sum_{i=1}^K \alpha_i^{(l+1)} f_i(z_s, \mu_i^{(l)}, (\sigma_i^2)^l, r_i)}$$

The updated equation of  $\sigma_i^2$  at  $(l+1)^{\text{th}}$  iteration is

$$\sigma_i^{2(l+1)} = \frac{\sum_{s=1}^N (z_s - \mu_i^{(l+1)})^2 \left( \frac{1}{2} - \frac{r_i (\sigma_i^2)^{(l)}}{\left( 2r_i \sigma_i^{2(l)} + (z_s - \mu_i^{(l+1)})^2 \right)^2} \right) t_i(z_s, \theta^{(l)})}{\sum_{s=1}^N t_i(z_s, \theta^{(l)})} \quad (9)$$

where,  $t_i(z_s, \theta^{(l)}) = \frac{\alpha_i^{(l+1)} f_i(z_s, \mu_i^{(l+1)}, (\sigma_i^2)^{(l)}, r_i)}{\sum_{i=1}^K \alpha_i^{(l+1)} f_i(z_s, \mu_i^{(l+1)}, (\sigma_i^2)^{(l)}, r_i)}$

#### 4. Initialization of the Parameters K – Means

The efficiency of the EM algorithm in estimating the parameters is heavily dependent on the number of regions in the image. The number of mixture components initially taken for K – Means algorithm is by plotting the histogram of the pixel intensities of the whole image. The number of peaks in the histogram can be taken as the initial value of the number of regions K.

The mixing parameters  $\alpha_i$  and the model parameters  $\mu_i, \sigma_i^2, r_i$  are usually considered as known apriori. A commonly used method in initializing parameters is by drawing a random sample from the entire image (McLachlan G and Peel D (2000)). This method performs well if the sample size is large and its computational time is heavily increased. When the sample size is small, some small regions may not be sampled. To overcome this problem we use a K – Means algorithm to divide the whole image into various homogeneous regions.

After determining the final values of K (number of regions), we obtain the initial estimates of  $\mu_i, \sigma_i^2, r_i$  and  $\alpha_i$  for the  $i^{\text{th}}$  region using the segmented region pixel intensities with the method given by Srinivasa Rao et al., (1997) for new symmetric distribution. The initial estimate  $\alpha_i$  is taken as  $\alpha_i = \frac{1}{K}$ , where,  $i=1,2,\dots,K$ . The shape parameter  $r_i$  can be estimated through sample kurtosis by using the following equation,

$$\left[ \frac{\frac{3\sqrt{\pi}}{4} + \sum_{j=1}^r \binom{r_i}{j} r_i^{-j} \left(j + \frac{1}{2}\right) \left(j + \frac{3}{2}\right) \Gamma\left(j + \frac{1}{2}\right)}{\left[ \frac{\sqrt{\pi}}{2} + \sum_{j=1}^r \binom{r_i}{j} r_i^{-j} \left(j + \frac{1}{2}\right) \Gamma\left(j + \frac{1}{2}\right) \right]^2} \right] \frac{(\pi)^{1/2} + \sum_{j=1}^r \binom{r_i}{j} r_i^{-j} \Gamma\left(j + \frac{1}{2}\right)}{\left[ \frac{\sqrt{\pi}}{2} + \sum_{j=1}^r \binom{r_i}{j} r_i^{-j} \left(j + \frac{1}{2}\right) \Gamma\left(j + \frac{1}{2}\right) \right]^2}$$

$$= \frac{\left[ \frac{1}{n} \sum_{i=1}^n (z_i - \bar{z})^4 \right]}{\left[ \frac{1}{n} \sum_{i=1}^n (z_i - \bar{z})^2 \right]^2}$$

By Rockies theorem there is one and only one real root to this equation. We can take the nearest integer to this real root as an estimate to the shape parameter  $r_i$ . Knowing the shape parameter  $r_i$  we can obtain the estimates of the parameter  $\mu_i$  and  $\sigma_i^2$  by method of moments as

$\mu_i = \bar{z}$  and

$$\sigma_i^2 = \frac{n}{(n-1)} \left[ 1 + \frac{\sum_{j=1}^{r_i} \binom{r_i}{j} r_i^{-j} \Gamma\left(j + \frac{1}{2}\right)}{2 \sum_{j=1}^{r_i} \binom{r_i}{j} r_i^{-j} \left(j + \frac{1}{2}\right) \Gamma\left(j + \frac{1}{2}\right)} \right] S^2$$

where,  $S^2$  is the sample variance.

#### 5. Segmentation Algorithm

In this section, we present the image segmentation algorithm. After refining the parameters the prime step in image segmentation is allocating the pixels to the segments of the image. This operation is performed by Segmentation Algorithm. The image segmentation algorithm consists of four steps.

**Step 1)** Plot the histogram of the whole image.

**Step 2)** Obtain the initial estimates of the model parameters using K-Means algorithm and moment estimators as discussed in section 3.

**Step 3)** Obtain the refined estimates of the model parameters by using the EM algorithm with the updated equations given by (7), (8) and (9) respectively.

**Step 4)** Assign each pixel into the corresponding  $j^{\text{th}}$  region (segment) according to Maximum likelihood of the  $j^{\text{th}}$  component ( $L_j$ ).

That is

$$L_j = \max_{j \in k} \left\{ \frac{\left( 2r + \left( \frac{z_s - \mu_j}{\sigma_j} \right)^2 \right)^r e^{-\frac{1}{2} \left( \frac{z_s - \mu_j}{\sigma_j} \right)^2}}{\sigma_j (2r)^r (2\pi)^{\frac{1}{2}} + \sum_{j=1}^r \binom{r}{j} (2r)^{r-j} 2^{j+\frac{1}{2}} \Gamma\left(j + \frac{1}{2}\right) \sigma_j} \right\}$$

$$-\infty < z_s < \infty, -\infty < \mu_j < \infty, \sigma_j > 0$$

#### 6. Experimental Results

To demonstrate the utility of the image segmentation algorithm developed in this chapter, an experiment is conducted with five images taken from Berkeley images dataset

(<http://www.eecs.berkeley.edu/Research/Projects/CS/Vision/bsds/BSDS300/html>). The images HORSE, MAN, BOAT

and TOWER are considered for image segmentation. The pixel intensities of the whole image are taken as feature. The pixel intensities of the image are assumed to follow a mixture of new symmetric distribution. That is, the image contains K regions and pixel intensities in each image region follows a new symmetric distribution with different parameters. The number of segments in each of the five images considered for experimentation is determined by the histogram of pixel intensities. The histograms of the pixel intensities of the five images are shown in figure 2.

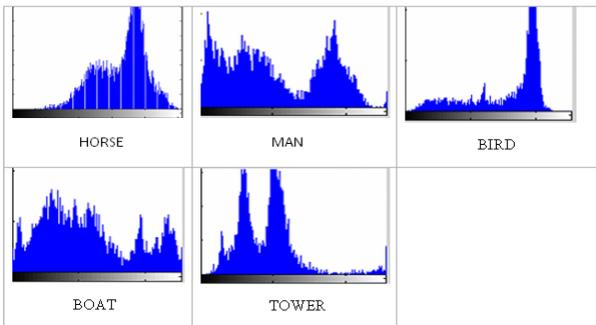


Figure 2: HISTOGRAMS OF THE IMAGES

The initial estimates of the number of the regions K in each image are obtained and given in Table1.

Table 1. INITIAL ESTIMATES OF K

| IMAGE         | HORSE | MAN | BIRD | BOAT | TOWER |
|---------------|-------|-----|------|------|-------|
| Estimate of K | 2     | 4   | 3    | 4    | 3     |

From Table 1, we observe that the image HORSE has two segments, images TOWER and BIRD have three segments each and images MAN and BOAT have four segments each. The initial values of the model parameters  $\mu_i, \sigma_i^2, r_i$  and  $\alpha_i$  for  $i = 1, 2, \dots, K$  for each image region are computed by the method given in section 3.

Using these initial estimates and the updated equations of the EM Algorithm given in Section 3 the final estimates of the model parameters for each image are obtained and presented in tables 2.a, 2.b, 2.c, 2.d and 2.e for different images.

| Table-2.a  |              |                               |  |                               |         |
|--|--------------|-------------------------------|--|-------------------------------|---------|
| ESTIMATED VALUES OF THE PARAMETERS FOR HORSE IMAGE |              |                               |  |                               |         |
| Estimation of Initial Parameters                   |              |                               | Estimation of Final Parameters by EM Algorithm |                               |         |
| Regions(i)   |              | Number of Image Regions (K=2) |  | Number of Image Regions (K=2) |         |
|  |              | 1                             | 2  | 1                             | 2       |
| Weights  | $\alpha_i$   | 0.5                           | 0.5  | 0.31361                       | 0.68639 |
| Means  | $\mu_i$      | 121.47                        | 187.91   | 145.01                        | 170.99  |
| Variances  | $\sigma_i^2$ | 609.82                        | 426.21   | 1943.1                        | 1034.3  |
| Estimated r values                                 | r            | 2                             | 1  | 2                             | 1       |

| Table-2.b  |              |                               |        |        |  |                               |        |         |          |
|--|--------------|-------------------------------|--------|--------|--|-------------------------------|--------|---------|----------|
| ESTIMATED VALUES OF THE PARAMETERS FOR MAN IMAGE |              |                               |        |        |  |                               |        |         |          |
| Estimation of Initial Parameters                 |              |                               |        |        | Estimation of Final Parameters by EM Algorithm |                               |        |         |          |
| Regions(i)                                       |              | Number of Image Regions (K=4) |        |        |  | Number of Image Regions (K=4) |        |         |          |
|  |              | 1                             | 2      | 3      | 4  | 1                             | 2      | 3       | 4        |
| Weights  | $\alpha_i$   | 1/4                           | 1/4    | 1/4    | 1/4  | 0.18275                       | 0.5234 | 0.23545 | 0.081981 |
| Means  | $\mu_i$      | 36.503                        | 75.342 | 126.54 | 203.31   | 64.46                         | 31.213 | 183.49  | 113.5    |
| Variances  | $\sigma_i^2$ | 298.16                        | 161.79 | 393.55 | 897.3  | 1549.3                        | 835.35 | 529.85  | 3104     |
| Estimated r values                               | $r_i$        | 4                             | 4      | 1      | 2  | 4                             | 4      | 1       | 2        |

| Table-2.d   |              |                               |       |        |  |                               |        |         |        |
|---|--------------|-------------------------------|-------|--------|--|-------------------------------|--------|---------|--------|
| ESTIMATED VALUES OF THE PARAMETERS FOR BOAT IMAGE |              |                               |       |        |  |                               |        |         |        |
| Estimation of Initial Parameters                  |              |                               |       |        | Estimation of Final Parameters by EM Algorithm |                               |        |         |        |
| Regions(i)  |              | Number of Image Regions (K=4) |       |        |  | Number of Image Regions (K=4) |        |         |        |
|   |              | 1                             | 2     | 3      | 4  | 1                             | 2      | 3       | 4      |
| Weights   | $\alpha_i$   | 1/4                           | 1/4   | 1/4    | 1/4  | 0.2540                        | 0.2670 | 0.23038 | 0.2485 |
| Means   | $\mu_i$      | 34.98                         | 81.14 | 131.13 | 216.5  | 37.644                        | 80.467 | 130.62  | 213.83 |
| Variances   | $\sigma_i^2$ | 342.0                         | 249.4 | 372.13 | 618.0  | 632.41                        | 565.83 | 684.41  | 775.17 |
| Estimated r values                                | $r_i$        | 5                             | 2     | 2      | 3  | 5                             | 2      | 2       | 3      |

| Table-2.c   |              |                               |        |  |                               |         |         |
|---|--------------|-------------------------------|--------|--|-------------------------------|---------|---------|
| ESTIMATED VALUES OF THE PARAMETERS FOR BIRD IMAGE |              |                               |        |  |                               |         |         |
| Estimation of Initial Parameters                  |              |                               |        | Estimation of Final Parameters by EM Algorithm |                               |         |         |
| Regions (i)                                       |              | Number of Image Regions (K=3) |        |  | Number of Image Regions (K=3) |         |         |
|   |              | 1                             | 2      | 3  | 1                             | 2       | 3       |
| Weights   | $\alpha_i$   | 1/3                           | 1/3    | 1/3  | 0.12722                       | 0.21391 | 0.65886 |
| Means   | $\mu_i$      | 53.491                        | 124.05 | 193.19   | 65.184                        | 127.31  | 192.79  |
| Variances   | $\sigma_i^2$ | 503.23                        | 43.05  | 148.42   | 1556.5                        | 2447.1  | 86.651  |
| Estimated r values                                | $r_i$        | 3                             | 3      | 1  | 3                             | 3       | 1       |

| Table-2.e  |              |                               |        |  |                               |        |          |
|--|--------------|-------------------------------|--------|--|-------------------------------|--------|----------|
| ESTIMATED VALUES OF THE PARAMETERS FOR TOWER IMAGE |              |                               |        |  |                               |        |          |
| Estimation of Initial Parameters                   |              |                               |        | Estimation of Final Parameters by EM Algorithm |                               |        |          |
| Regions (i)  |              | Number of Image Regions (K=3) |        |  | Number of Image Regions (K=3) |        |          |
|  |              | 1                             | 2      | 3  | 1                             | 2      | 3        |
| Weights  | $\alpha_i$   | 1/3                           | 1/3    | 1/3  | 0.71336                       | 0.1941 | 0.092544 |
| Means  | $\mu_i$      | 64.363                        | 107.54 | 185.83   | 71.14                         | 110.03 | 145.58   |
| Variances  | $\sigma_i^2$ | 358.2                         | 393.11 | 1253.6   | 602.94                        | 3140   | 2975.9   |
| Estimated r values                                 | $r_i$        | 3                             | 2      | 3  | 3                             | 2      | 3        |

Substituting the final estimates of the model parameters, the probability density function of pixel intensities of each image is estimated. Using the estimated probability density functions and the image segmentation algorithm given in section 5, the image segmentation is done for each of the five images under consideration. The original and segmented images are shown in figure 3

Figure 3 Original and segmented Images

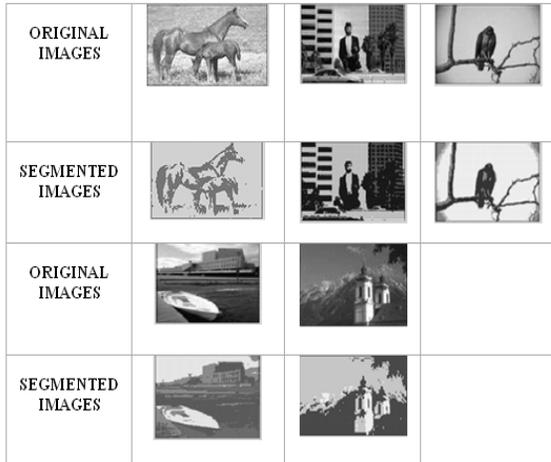


Table 3: SEGMENTATION PERFORMANCE MEASURES

| IMAGES | METHOD | PERFORMANCE MEASURES |        |        |
|--------|--------|----------------------|--------|--------|
|        |        | PRI                  | GCE    | VOI    |
| HORSE  | FGMM   | 0.9403               | 0.8827 | 9.4277 |
|        | FNSDMM | 0.9321               | 0.8759 | 9.3763 |
| MAN    | FGMM   | 0.9779               | 0.9111 | 9.1134 |
|        | FNSDMM | 0.9727               | 0.9069 | 9.3946 |
| BIRD   | FGMM   | 0.8734               | 0.7273 | 7.5725 |
|        | FNSDMM | 0.8358               | 0.6809 | 7.4998 |
| BOAT   | FGMM   | 0.9765               | 0.8826 | 8.8952 |
|        | FNSDMM | 0.9713               | 0.8831 | 9.1621 |
| TOWER  | FGMM   | 0.9010               | 0.7308 | 8.0586 |
|        | FNSDMM | 0.9020               | 0.7421 | 8.2153 |

## 7. Performance Evaluation

After conducting the experiment with the image segmentation algorithm developed in this chapter, its performance is studied. The performance evaluation of the segmentation technique is carried by obtaining the four performance measures namely, (i) Probabilistic Rand Index (PRI), (ii) Variation Of Information (VOI) and (iii) Global Consistency Error (GCE). The Rand index given by Unnikrishnan et al (2005) counts the fraction of pairs of pixels whose labeling are consistent between the computed segmentation and the ground truth. This quantitative measure is easily extended to the Probabilistic Rand index (PRI) given by Unnikrishnan and et al (2007). The variation of information (VOI) metric given by Meila (2005) is based on relationship between a point and its cluster. It uses mutual information metric and entropy to approximate the distance between two clustering across the lattice of possible clustering. It measures the amount of information that is lost or gained in changing from one clustering to another. The Global Consistency Error (GCE) given by D.Martin and et al (2001) measures the extent to which one segmentation map can be viewed as a refinement of segmentation. For a perfect match, every region in one of the segmentations must be identical to, or a refinement (i.e., a subset) of, a region in the other segmentation.

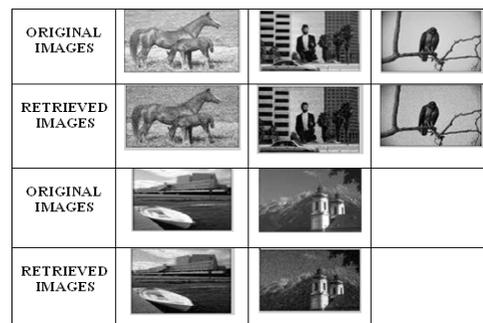
The performance of developed algorithm using finite new symmetric distribution mixture model (FNSDMM) is studied by computing the segmentation performance measures namely, PRI, GCE and VOI for the five images under study. The computed values of the performance measures for the developed algorithm and the earlier existing Finite Gaussian Mixture Model (FGMM) with K-Means algorithm are presented in table 3 for a comparative study.

From table 3 it is observed that the PRI values of the proposed algorithm for the five images considered for experimentation are less than that of the values from the segmentation algorithm based on Finite Gaussian Mixture Model with K-means. Similarly GCE and VOI values of the proposed algorithm are less than that of Finite Gaussian Mixture Model. This reveals that the proposed algorithm outperforms the existing algorithm based on the Finite Gaussian Mixture Model. When the kurtosis parameter of each component of the model is zero, the model reduces to Finite Gaussian Mixture Model and even in this case the algorithm performs well.

After developing the image segmentation method it is needed to verify the utility of segmentation in the model building of the image for image retrieval. The performance evaluation of the retrieved image can be done by subjective image quality testing or by objective image quality testing. The objective image quality testing methods are often used since the numerical results of an objective measure allow a consistent comparison of different algorithms. There are several image quality measures available for performance evaluation of the image segmentation method. An extensive survey of quality measures is given by Eskicioglu A.M. and Fisher P.S. (1995). For the performance evaluation of the developed segmentation algorithm, we consider the image quality measures a) Average Difference, b) Maximum Distance, c) Image Fidelity, d) Mean Square Error, e) Signal to Noise Ratio and f) Image Quality Index.

Using the estimated probability density functions of the images under consideration the retrieved images are obtained and are shown in figure 4.

Figure 4: The Original and Retrieved Images



The image quality measures are computed for the five retrieved images HORSE, MAN, BIRD, BOAT and TOWER using the proposed model and FGMM with K-means and their values are given in the table 4.

Table 4: Comparative Study of Image Quality Metrics

| IMAGE | Quality Metrics       | FGMM   | FNSDMM with K-Means | Standard Limits    |
|-------|-----------------------|--------|---------------------|--------------------|
| HORSE | Average Difference    | 0.5011 | 0.4089              | Close to 1         |
|       | Maximum Distance      | 1.0000 | 1.0000              | Close to 1         |
|       | Image Fidelity        | 1.0000 | 1.0000              | Close to 1         |
|       | Mean Square Error     | 0.5011 | 0.4090              | Close to 0         |
|       | Signal to Noise Ratio | 5.6542 | 6.0957              | As big as possible |
|       | Image Quality Index   | 1.0000 | 1.0000              | Close to 1         |
| MAN   | Average Difference    | 0.4858 | 0.4907              | Close to 1         |
|       | Maximum Distance      | 1.0000 | 1.0000              | Close to 1         |
|       | Image Fidelity        | 1.0000 | 1.0000              | Close to 1         |
|       | Mean Square Error     | 0.4946 | 0.4995              | Close to 0         |
|       | Signal to Noise Ratio | 5.6828 | 5.6615              | As big as possible |
|       | Image Quality Index   | 1.0000 | 1.0000              | Close to 1         |
| BIRD  | Average Difference    | 0.4939 | 0.5050              | Close to 1         |
|       | Maximum Distance      | 1.0000 | 1.0000              | Close to 1         |
|       | Image Fidelity        | 1.0000 | 1.0000              | Close to 1         |
|       | Mean Square Error     | 0.5050 | 0.4939              | Close to 0         |
|       | Signal to Noise Ratio | 5.6861 | 5.6376              | As big as possible |
|       | Image Quality Index   | 1      | 1.0000              | Close to 1         |
| BOAT  | Average Difference    | 0.5039 | 0.5043              | Close to 1         |
|       | Maximum Distance      | 1.0000 | 1.0000              | Close to 1         |
|       | Image Fidelity        | 1.0000 | 1.0000              | Close to 1         |
|       | Mean Square Error     | 0.5070 | 0.5064              | Close to 0         |
|       | Signal to Noise Ratio | 5.6318 | 5.6291              | As big as possible |
|       | Image Quality Index   | 1      | 1.0000              | Close to 1         |
| TOWER | Average Difference    | 0.4936 | 0.5074              | Close to 1         |
|       | Maximum Distance      | 1.0000 | 1.0000              | Close to 1         |
|       | Image Fidelity        | 0.9999 | 0.9999              | Close to 1         |
|       | Mean Square Error     | 0.5076 | 0.4936              | Close to 0         |
|       | Signal to Noise Ratio | 5.6870 | 5.6264              | As big as possible |
|       | Image Quality Index   | 1.0000 | 1.0000              | Close to 1         |

From the Table 4, it is observed that all the image quality measures for the five images are meeting the standard criteria. This implies that using the proposed algorithm the images are retrieved accurately. A comparative of study of proposed algorithm with that of algorithm based on Finite Gaussian Mixture Model reveals that the MSE of the proposed model is less than that of the Finite Gaussian Mixture Model. Based on all other quality metrics also it is observed that the performance of the proposed model in retrieving the images is better than the Finite Gaussian Mixture Model.

## 8. Conclusions

In this paper we propose an unsupervised image segmentation algorithm based on finite new symmetric mixture model with K-means clustering. The finite mixture of new symmetric distribution includes Finite

Gaussian Mixture Model as a particular case when the kurtosis parameter equals to zero. This includes several platy-kurtic mixture distributions as particular cases. As a result of this generic nature this algorithm can handle a wide variety of images. An EM algorithm is developed and used for estimating the model parameters. In our experimentation with five images taken from Berkeley image data set, it is observed that the developed algorithm performs better with respect to the image segmentation metrics and the image quality metrics. The hybridization of model based approach with K-means has improved the accuracy of retrieval. This algorithm can be utilized for image analysis and retrieval of grey and colour images more accurately.

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