Fractional Identification of Rotor Skin Effect in Induction Machines

Abdelhamid JALLOUL[1], Khaled JELASSI[1], Pierre MELCHIOR[2], Jean-Claude TRIGEASSOU[2]

[1]Laboratoire des Systèmes Electriques (LSE), Ecole Nationale d'Ingénieurs de Tunis, B.P.37 le Belvédère 1002 Tunis Tunisie

[2]Laboratoire Intégration du Matériau au Système (IMS-LAPS), UMR 5218, Université Bordeaux 1, France

Abstract

Fractional identification of rotor skin effect in induction machines is presented in this paper. Park's transformation is used to obtain a system of differential equations which allows to include the skin effect in the rotor bars of asynchronous machines. A transfer function with a fractional derivative order has been selected to represent the admittance of the bar by the help of a non integer integrator which is approximated by a J+1 dimensional modal system. The machine parameters are estimated by an output-error technique using a non linear iterative optimization algorithm. Numerical simulations and experimental results show the performance of the modal approach for modeling and identification.

Keywords: *Induction machines, Park's transformation, skin effect, fractional impedance, ladder network, non integer order differentiation, output error identification.*

1. Introduction

Accurate modeling of electrical machines is very important for the designer of the machine, facing its improvement. On the other hand, the knowledge of parameters is necessary to realize realistic simulations of the machines and is important for the operator of modern drives who implements control systems.

Moreover, in the case of the association of a static converter to an electrical machine, the rational use of the whole passes by a perfect control of the global dynamic behavior. With PWM power supplies, the electrical machines have to work on a very large frequency range. Thus, the representation of this machine by a simplified model, only valid on a limited frequency range, is the source of unsatisfactory results.
The insufficiency of these models is more accentuated when the electrical machines have a massive structure (like asynchronous machines with cages, deep notches or massive rotor) characterized by skin effect (or frequency effect).

Induction currents in the rotor bars are governed by a diffusive phenomenon. At low frequencies, currents have a density which is uniform and equal everywhere over the entire cross sectional area. If the frequency is high enough, current density tends to be higher at the surface of the bar. The higher the frequency, the greater the tendency for this effect to occur. This phenomenon is called «skin effect» in rotor bars, or «frequency effect» more generally.
There are three possible reasons we might care about skin effect [1]:
- The skin effect causes the effective cross sectional area to decrease. Therefore, the skin effect causes the effective resistance of the conductor to increase.
- The skin effect is a function of frequency. Therefore, the skin effect causes the resistance of a conductor to become a function of frequency.
- If the skin effect causes the effective cross sectional area of a bar to decrease and its resistance to increase, then the bar will heat faster and to a higher temperature at higher frequencies for the same level of current.

In electrical engineering, this phenomenon is particularly important in massive rotor or squirrel-cage induction motors. Its diffusive character leads to notice a strong modification of the impedance (both resistance and reactance) according to the frequency [2]. It is thus interesting to use a transfer function with a fractional derivative order to represent the admittance of the bar on a broad frequency scale, like it has been demonstrated in a recent paper [13].
In the context of parameter estimation of the admittance, the derivative orders should be estimated in the same way

that the other coefficients. Based on the output error method, the models used are non linear in the parameters and optimization algorithms involve non linear programming (NLP).

In this paper, we propose to identify the parameters of the asynchronous machine model taking into account the fractional feature of the rotor model and estimating its parameters using an output error identification technique.
After a reminder of definitions related to fractional integration operators in parts 2 and 3, the Park's model of asynchronous machines with fractional impedance is presented in parts 4. Part 5 is devoted to present the output error method. We propose, in part 6 an application of fractional identification techniques for parameter estimation. A comparison and a discussion of identification techniques and experimental results are proposed in parts 6 and 7.

2. Fractional Differentiation and Integration

Fractional integration is defined by the Riemann-Liouville Integral [20][21][22][6].
The nth order integral (n real positive) of the function $f(t)$ is defined by the relation:

$$I_n(f(t)) = \frac{1}{\Gamma(n)}\int_0^t (t-\tau)^{n-1} f(\tau)d\tau \quad (1)$$

where $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$ is the gamma function.

$I_n(f(t))$ is interpreted as the convolution [20] of the function $f(t)$ with the impulse response:

$$h_n(t) = \frac{t^{n-1}}{\Gamma(n)} \quad (2)$$

of the fractional integration operator whose Laplace transform is:

$$I_n(s) = L\{h_n(t)\} = \frac{1}{s^n} \quad (3)$$

Fractional differentiation is the dual operation of the fractional integration.
Consider the fractional integration operator $I_n(s)$ whose input/output are respectively $x(t)$ and $y(t)$.
Then

$$y(t) = I_n(x(t)) \quad (4)$$

or

$$Y(s) = \frac{1}{s^n}X(s) \quad (5)$$

Reciprocally, $x(t)$ is the nth order fractional derivative of $y(t)$ defined as:

$$x(t) = D_n(y(t)) \quad (6)$$

or

$$X(s) = s^n\,Y(s) \quad (7)$$

where s^n represents the Laplace transform of the fractional differentiation operator (with zero initial conditions).

3. Fractional Integration Operators

The fractional integration operator $I_n(s)$ is the key element for FDE simulation. However, the realization of $I_n(s)$ is not a simple problem as in the integer order case. It is possible to consider the frequency and modal approaches. Our objective is to compare the impact of these approaches for the simulation and the identification of the asynchronous machine.

3.1 Frequency Approach Synthesis

3.1.1 Principle:

Let us consider the Bode plots of a fractional integrator truncated in low and high frequencies (Fig. 1) [3][4][5].

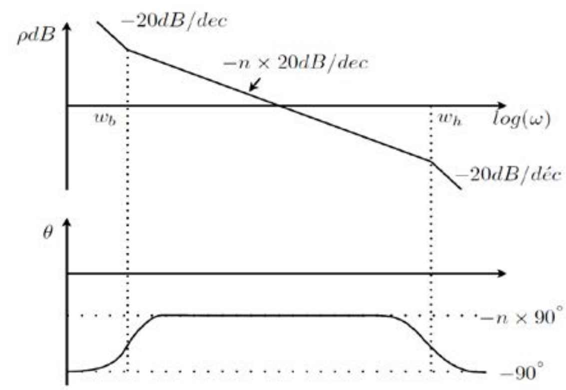

Fig. 1. Bode diagram of the fractional integrator

It is composed of three parts. The intermediary part corresponds to non-integer action, characterized by the order n. In the two other parts, the integrator has a conventional action, characterized by its order equal to 1.

In this way, the operator $\widetilde{I}_n(s)$ is defined as a conventional integrator, except in a limited band $[w_b; w_h]$ where it acts like s^{-n}. The operator $\widetilde{I}_n(s)$ is defined using a fractional phase-lead filter [6] and an integrator s^{-n}.

$$\tilde{I}_n(s) = \frac{G_n}{s} \prod_{j=1}^{J} \frac{1+\frac{s}{w'_j}}{1+\frac{s}{w_j}} \tag{8}$$

The coefficient *Gn* is a normalized factor, such as $\tilde{I}_n(s)$ and $I_n(s)$ are identical ou $[w_b; w_h]$.

This operator is completely defined by the following relations [6]:

$$w_j = \alpha\, w'_j, \tag{9}$$

$$w'_{j+1} = \eta\, w_j, \tag{10}$$

$$n = 1 - \frac{\log \alpha}{\log \alpha\eta} \tag{11}$$

α and η are recursive parameters related to the non integer order n.
When J is sufficiently large, the bode diagram of $\tilde{I}_n(s)$ tends towards the ideal one of Fig.1.

3.1.2 State space model of $\tilde{I}_n(s)$

It is convenient to associate a state-space representation to $\tilde{I}_n(s)$ in order to simulate fractional systems. There is an infinite number of possibilities to represent $\tilde{I}_n(s)$ by a state space model. Practically, we have chosen the one where the state variables correspond to the outputs of the elementary cells of $\tilde{I}_n(s)$.

$$X_j(s) = \frac{1+\frac{s}{w'_j}}{1+\frac{s}{w_j}} X_{j-1}(s) \tag{12}$$

or

$$-\alpha\, \dot{x}_{j-1} + \dot{x}_j = w_j (x_{j-1} - x_j) \text{ for } j = 1 \text{ to } J \tag{13}$$

with $X_0 = \frac{G_n}{s} V(s)$.

where *v(t)* is the input of $\tilde{I}_n(s)$ and $x_J(t) = x(t)$ its output.
The corresponding state space model is:

$$M_I\, \underline{\dot{x}}_I(t) = A_I\, \underline{x}_I(t) + \underline{B}_I v(t) \tag{14}$$

with

$$M_I = \begin{bmatrix} 1 & & & & 0 \\ -\alpha & 1 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ 0 & & & -\alpha & 1 \end{bmatrix}$$

$$A_I = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ w_1 & -w_1 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & w_J & -w_J \end{bmatrix} \quad \underline{B}_I = \begin{bmatrix} G_n \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}; \ \underline{x}_I = \begin{bmatrix} x_0 \\ \vdots \\ x_j \\ \vdots \\ x_J \end{bmatrix}$$

Because one of our objectives is to estimate the parameters we have privileged parsimonious models in order to facilitate the identification procedure [4][7]. Other approximations can be used and bring improvements to simplify the calculations of the frequency domain approach, like the modal approach.

3.2 Modal Approach

3.2.1 Frequency distributed model

The fractional order integrator is a linear system such as

$$y(t) = h(t) * u(t) \tag{15}$$

with

$$H(s) = \frac{1}{s^n} = L\{h(t)\} \quad 0 < n < 1 \tag{16}$$

This system can be represented by a frequency distributed model; it is also known as a diffusive model (refer to appendix 1 and [8] [9] [10]):

$$\begin{cases} \frac{\partial x(t,w)}{\partial t} = -\omega\, x(w,t) + u(t) \\ y(t) = \int_0^\infty \mu(\omega) x(w,t) dw \end{cases} \tag{17}$$

and

$$h(t) = \int_0^\infty \mu(w)\, x(w,t)\, dw \tag{18}$$

with

$$\mu(w) = \frac{\sin(n\pi)}{\pi} w^{-n}, \ 0 < n < 1 \tag{19}$$

$\mu(w)$ is called frequency weighting function.

3.2.2 Frequency discretized distributed model

This continuous frequency weighted model is not directly usable. A practical model (necessary for simulation applications) is obtained by frequency discretization of $\mu(w)$, where the function $\mu(w)$ is replaced by a multiple step function (with K steps). For an elementary step, its height is $\mu(w_k)$, and its width is Δw_k. Let c_k be the weight of the k[th] element:

$$c_k = \mu(w_k)\Delta w_k \tag{20}$$

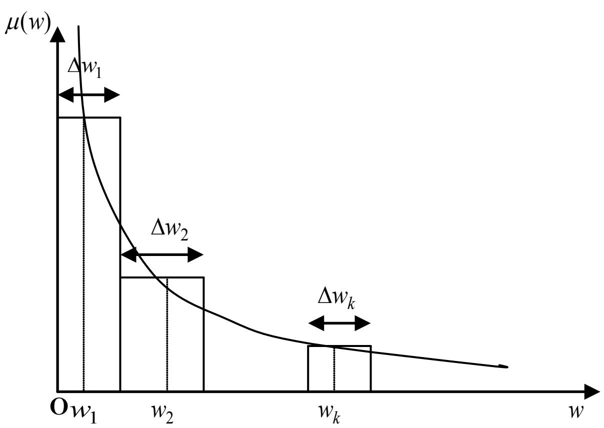

Fig. 2. Frequency discretization of $\mu(w)$

Thus, the continuous distributed model becomes a conventional state model with dimension equal to K.

$$\begin{cases} \dfrac{dx_k}{dt} = -w_k x_k(t) + u(t); \quad k = 1..K \\ y(t) = \displaystyle\sum_{k=1}^{K} \mu(w_k) x_k(t) \Delta w_k \\ \quad = \displaystyle\sum_{k=1}^{K} c_k x_k(t) \end{cases} \tag{21}$$

or equivalently:

$$\begin{cases} \underline{\dot{X}}(t) = A\underline{X}(t) + \underline{B}u(t) \\ y(t) = \underline{C}^T \underline{X}(t) \end{cases} \tag{22}$$

with

$$\underline{X}(t) = \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix} \quad A = \begin{bmatrix} -w_1 & & 0 \\ & \ddots & \\ 0 & & -w_k \end{bmatrix};$$

$$\underline{B}^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}, \ \underline{\underline{C}}^T = \begin{bmatrix} c_1 & \cdots & c_K \end{bmatrix}$$

With this approach, we obtain a discrete state-space model which is frequency distributed with the constraints:

$w_1 \to 0$, $w_K \to \infty$ and $K >> 1$

3.3 Comparison with Frequency Model

It is easy to transform the model (14) of $I_n(s)$ into a modal form because the w_j are known a priori. This transformation is based on the following decomposition in simple elements:

$$\tilde{I}_n(s) = \frac{c_0}{s} + \sum_{j=1}^{J} \frac{c_j}{s + w_j} \tag{23}$$

where c_0 and c_j coefficients are linked to G_n, w_j and $w_j^{'}$ by the relations:

$$c_0 = G_n \tag{24}$$

$$c_j = G_n \frac{w_j - w_j^{'}}{w_j^{'}} \prod_{\substack{i=1 \\ i \neq j}}^{J} \frac{1 - \dfrac{w_j}{w_i^{'}}}{1 - \dfrac{w_j}{w_i}} \tag{25}$$

This second definition of $\tilde{I}_n(s)$ corresponds to a modal state model:

$$\begin{cases} \underline{\dot{X}}^{'}(t) = A_I^{'} \underline{X}_I^{'}(t) + \underline{B}_I^{'}(t)u(t) \\ y(t) = \underline{C}_I^T \underline{X}^{'}(t) \end{cases} \tag{26}$$

with:

$$A_I^{'} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & -w_1 & & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -w_J \end{bmatrix}; \ \underline{X}^{'}(t) = \begin{bmatrix} x_0^{'} \\ x_1^{'} \\ \vdots \\ x_J^{'} \end{bmatrix}; \ \underline{B}_I^{'} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\underline{C}_I^T = \begin{bmatrix} c_0 & c_1 & \cdots & c_J \end{bmatrix}$$

In the frequency domain approach, the modes w_j are indirectly obtained by $\tilde{I}_n(s)$ in the $[w_b; w_h]$ interval, they correspond to the modes of the modal approach. The interest of this last representation is that the modes are decoupled, which allows fast computations. Moreover, an important interest of $w_0 = 0$ is to reject static error in simulation applications.

4. Park's Model of Induction Machine

The most important assumptions to derive the Park's model are:

- The air gap between the stator magnetic structure and the rotor magnetic structure is uniform. All magnetic variations due to slots are neglected.
- The magnetic field is assumed to have a sinusoidal spatial distribution.
- The stator and rotor windings axes coincide with the magnetic axes of the phases.
- The permeability of the iron is infinite.

The Park's transformation establishes an equivalence between a three-phase representation and rotor reference frame.

The conventional equivalent diagram [11] of Park's model is represented on Fig.4 :

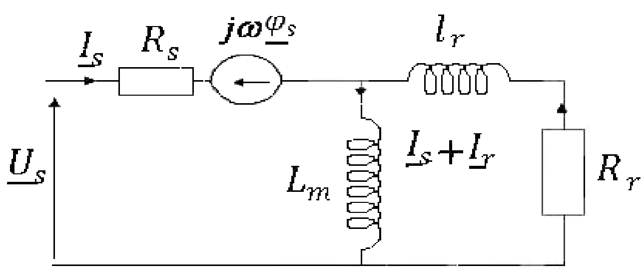

Fig.3. Conventional equivalent diagram of Park's model

with:

R_s and R_r representing the resistance of the stator and the resistance of the rotor bars respectively

ω is the rotor speed, l_r are the stator and rotor leakage inductances, L_m the magnetizing inductance.

4.1 Ladder Model

To take into account skin effect in rotor bars, the assumption is made that each equivalent rotor winding is composed of K slices in parallel.

The Park's equivalent diagram with ladder model (refer to [12] and [13] for more details) is represented on Fig.5

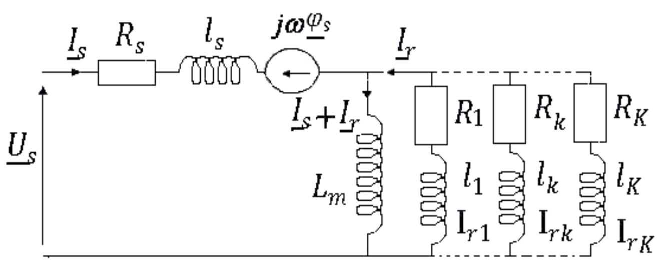

Fig. 4. Park's ladder model

l_k represents the linkage inductance of each elementary slice.

A complex notation is used:

$$x_d + jx_q = \underline{X} \tag{27}$$

The mathematical model of squirrel cage induction motor can be written as:

$$\begin{cases} \underline{U}_s = R_s \underline{I}_s + \dfrac{d}{dt}\underline{\varphi}_s + j\omega\underline{\varphi}_s \\ 0 = R_r \underline{I}_r + \dfrac{d}{dt}\underline{\varphi}_r \\ \underline{\varphi}_s = l_s \underline{I}_s + L_m(\underline{I}_s + \underline{I}_r) \\ \underline{\varphi}_r = L_m(\underline{I}_s + \underline{I}_r) + \sum_{k=1}^{K} l_k \underline{I}_{r_k} \end{cases} \tag{28}$$

There are several expressions that can describe the developed electromechanical torque of an induction machine [11] [13], we prefer to use the following because it refers only to stator variables:

$$C_{em} = \varphi_{ds} i_{qs} - \varphi_{qs} i_{ds} \tag{29}$$

4.2 Induction machine equivalence with fractional impedance

Using equivalence between a ladder network and fractional impedance [13], one can define the Park's fractional model of the induction machine:

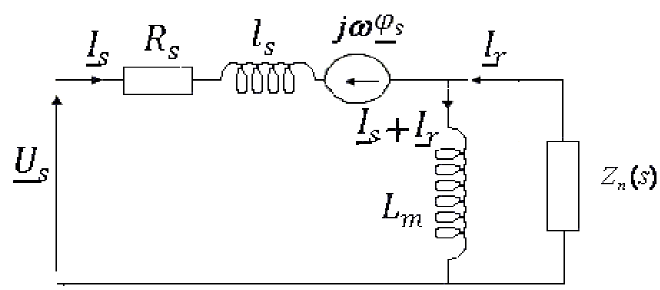

Fig.5. Park's fractional model

The equations describing electromagnetic processes in induction machine (including a squirrel-cage rotor) are as follows:

$$\begin{cases} \underline{U}_s = R_s \underline{I}_s + \dfrac{d}{dt}\underline{\varphi}_s + j\omega\underline{\varphi}_s \\ \underline{\varphi}_s = (L_m + l_s)\underline{I}_s + L_m \underline{I}_s \end{cases} \tag{30}$$

We define the magnetizing flux $\underline{\varphi}_m$

$$\underline{\varphi}_m = L_m(\underline{I}_s + \underline{I}_r) \tag{31}$$

we can write

$$s\underline{\varphi}_m = -I_r(s)Z_n(s) \quad (32)$$

then

$$s\underline{\varphi}_m(s) = -(\frac{a_0}{b_0}\underline{I}_r(s) + \frac{1}{b_0}s^n\underline{I}_r(s)) \quad (33)$$

which corresponds to the fractional order differential equation :

$$\frac{d}{dt}\underline{\varphi}_m(t) = -\frac{a_0}{b_0}\underline{I}_r(t) - \frac{1}{b_0}D_n(\underline{I}_r(t)) \quad (34)$$

because

$$\underline{\varphi}_s = \underline{\varphi}_m + l_s\underline{I}_s \quad (35)$$

we obtain a differential system allowing the simulation of the asynchronous machine:

$$\begin{cases} \frac{d}{dt}\underline{\varphi}_s = \underline{U}_s - R_s\underline{I}_s - j\omega\underline{\varphi}_s \\ D_n(\underline{I}_r) = -a_0\underline{I}_r - b_0(\underline{U}_s - R_s\underline{I}_s - l_s\frac{d}{dt}\underline{I}_s - j\omega\underline{\varphi}_s) \\ \underline{I}_s = \frac{\underline{\varphi}_s - L_m\underline{I}_r}{L_m + l_s} \end{cases} \quad (36)$$

The mechanical expression of the rotor speed is obtained thanks to the relation:

$$J\frac{d}{dt} = C_{em} - C_r - f\omega \quad (37)$$

C_{em} is expressed in (29), J : moment of inertia

f : friction coefficient

5. Output Error Identification Method

Next, we remind the principle of a method allowing the estimation of the parameters of the Park model of induction machine with fractional impedance (36).

Whereas parametric estimation can be performed by a linear optimization technique in case [14] the model is linear in the parameters, the estimation of the derivative orders and of the coefficients requires the use of a nonlinear programming algorithm.

The method suggested by Trigeassou, Lin and Poinot [3] [7], is based on the definition of a non integer integration operator limited in frequency (frequency approach).

The model of the system is in continuous time representation and we use an output error technique (OE) to estimate its parameters [15] [16].

For the fractional state-space model of the induction machine, the parameter vector is defined by:

$$\underline{\theta}^T = \begin{bmatrix} R_s & L_m & a_0 & b_0 & n \end{bmatrix} \quad (38)$$

The state-space model is simulated using a n umerical integration algorithm, thus one gets:

$$\hat{\underline{I}}_{s_i} = f_i(u, \hat{\underline{\theta}}_i) \quad (39)$$

where $\hat{\underline{\theta}}_i$ is an estimation of $\underline{\theta}$ at iteration i.

The optimal value of $\hat{\underline{\theta}}(\underline{\theta}_{opt})$ is obtained by minimization of the quadratic criterion:

$$J = \sum_{k=1}^{K}(i_{d_s}^* - \hat{i}_{d_s})^2 + \sum_{k=1}^{K}(i_{q_s}^* - \hat{i}_{q_s})^2 \quad (40)$$

we obtain:

$$\hat{\underline{\theta}}_{i+1} = \hat{\underline{\theta}}_i + \Delta\underline{\theta} \quad (41)$$

where $\Delta\underline{\theta}$ depends on the optimization algorithm.

We can use a black box technique provided by the Matlab toolbox functions in order to minimize J. In this case we want to obtain the optimal $\underline{\theta}_{opt}$ without worrying of how we obtain this estimate. But this technique presents some drawbacks such as the absence of direct informations on the criterion at the optimum, thus in particular on the precision (sensitivity of J with regard to the different estimates).

To remedy these drawbacks, we use sensitivity functions of the simulated output [15] [16].

Because $\hat{\underline{I}}_s(t)$ is non linear in $\hat{\underline{\theta}}$, a N on Linear Programming technique is used to estimate iteratively $\hat{\underline{\theta}}_i$:

$$\hat{\underline{\theta}}_{i+1} = \hat{\underline{\theta}}_i - \left\{ \left[J''_{\theta\theta} + \lambda I \right]^{-1} \underline{J}'_\theta \right\}_{\hat{\theta}=\theta_i} \quad (42)$$

with [15] [16]:

$$\begin{cases} \underline{J}'_\theta = -2\sum_{k=1}^{K}\varepsilon_k\underline{\sigma}_{k,\underline{\theta}_i} : gradient \\ J''_{\theta\theta} \approx 2\sum_{k=1}^{K}\underline{\sigma}_{k,\underline{\theta}_i} \quad : \text{hessien} \\ \lambda : \text{Marquardt parameter} \\ \underline{\sigma}_{k,\underline{\theta}_i} = \frac{\partial\hat{y}_k}{\partial\underline{\theta}_i} : \text{sensitivity function} \end{cases} \quad (43)$$

This algorithm, known as Marquardt's one [17], often used in non linear optimization, ensures robust convergence in spite of a bad initialization of $\hat{\underline{\theta}}$. A good precision of the output sensibility functions $\underline{\sigma}_{k,\underline{\theta}_i}$ is however necessary to ensure a good convergence and precision of the algorithm.

6. Numerical Simulations

In order to compare the previous identification techniques, an application on asynchronous machine is treated to exhibit the performances of each approach.

The data set is composed of K data pairs $\{u_k, y_k^*\}$ with $t = k\,T_e$ (T_e : sampling period).

The simulations are performed with Matlab software.

The integrator parameters used for simulation are: $\omega_1 = 10^{-4}\,rd/s$, $\omega_J = 10^4\,rd/s$ cells number $J = 20$ $T_e = 2.7e^{-5}\,s$, $n = 0.6$.

The machine parameters are:

$R_s = 9.810, L_m = 0.4360, a_0 = 50.2625, b_0 = 13.1224.$

The input/output data for identification are provided by a simulation of the asynchronous machine starting.

6.1 Identification using Matlab Toolbox

First, the identification of the machine parameters has been performed by the direct approach using Matlab toolbox functions. We use the function LSQNONLIN which is used to solve non-linear least squares problems.

This function minimizes the sum of squares of the functions with the default optimization parameters replaced by values in the structure OPTIONS, an argument created with the OPTIMSET function.

We obtain the results on figure 5:

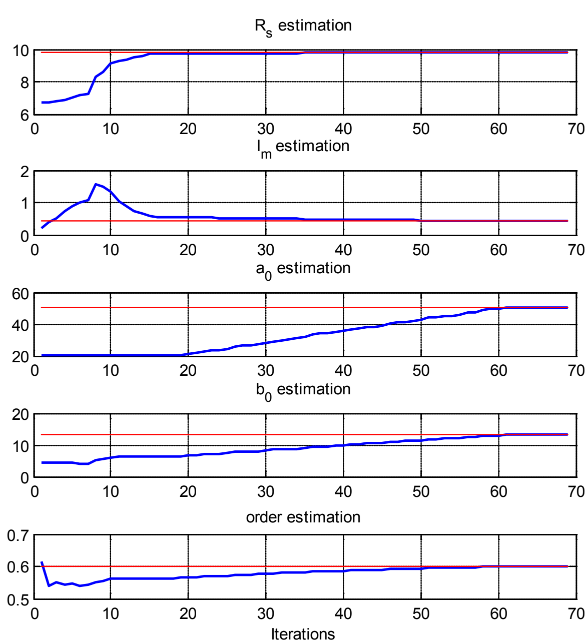

Fig. 6. Matlab toolbox identification method

6.2 Identification using frequency domain approach

In this section, we present the identification results on figure 6 which have been performed by the frequency approach (for the fractional integration operator).

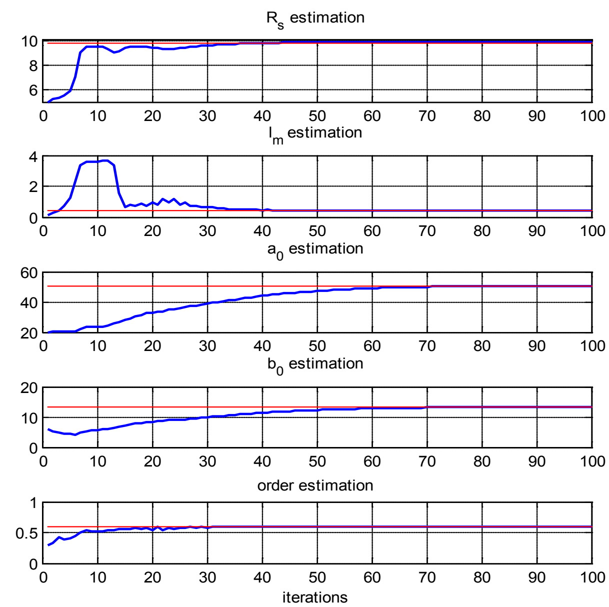

Fig. 7. Identification by the "frequency domain" approach

This method is based on the simulation of the sensitivity functions [18] using the output error method. It gives faster convergence than the direct approach, but it leads to an important calculation load.

Moreover, the analytical calculation of sensitivity functions can be inextricable, even unnecessarily complex, concerning the output sensitivity of the parameter n (with regard to the coefficients: α, η). For this reason we prefer now to use the modal model of the fractional integrator.

6.3 Identification using modal approach

The modal formulation is not adapted to the exact calculation of $\frac{\partial x(t)}{\partial n}$ because the ω_k and c_k are complicated functions of n. It is possible to simplify and proceed directly the calculation of the sensitivity functions [15] [16] [19] by numerical differentiation, in the form:

$$\frac{\partial x(\hat{n},t)}{\partial \hat{n}} = \lim_{\Delta n \to 0} \frac{x(\hat{n}+\Delta n,t) - x(\hat{n},t)}{\Delta n} \tag{44}$$

A preliminary study is essential for the choice of Δn. In the general case, $\Delta\theta$ is difficult to choose because θ can vary from $-\infty$ to ∞. Because $0 < n < 1$, it is easy to find an optimal value of Δn, which will be always the same. Then the calculation becomes more simple.

We have represented in figure 7 the identification results using the modal representation.

As exhibited by theses graphs, convergence is very quick.

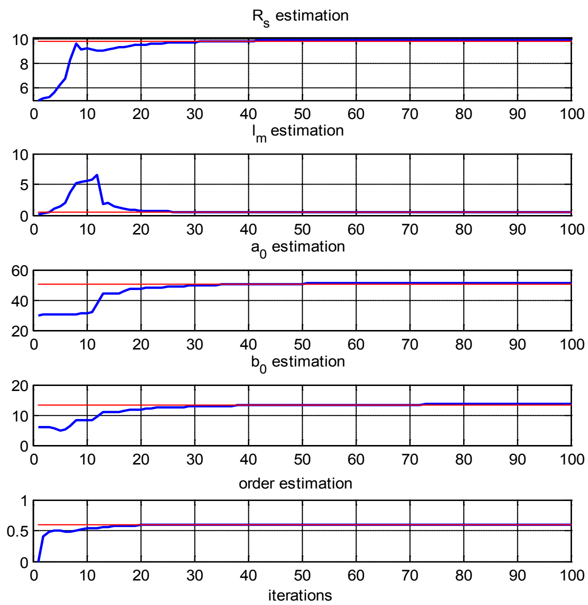

Fig. 8. Identification by modal approach

Moreover, this technique provides results (refer to table 1) with good precision

6.4 Comparison of the Methods

The use of the Matlab toolbox identification technique is simple, but its presents some defects such as the absence of direct informations on the criterion at the optimum. Moreover, the convergence appears to be very slow.

The method of the fractional integrator (with frequency approach) is more complex to implement. However, it relies on a state-space representation allowing to generalize the fundamental concepts related to ODEs.

Finally, the use of the modal representation of the fractional integrator reduces the computation time compared to poles/zeros approach and its programming is much simple, which is an important feature in the context of complex systems.

We present in the following table the optimal parameters $\underline{\theta}_{opt}$ after the convergence for each approach:

Table 1. Optimal parameters results

Techniques / Exact parameters		Matlab Toolbox	Frequency approach	Modal approach
R_s	9.81	9.810	9.8987	9.8103
L_m	0.4360	0.436	0.3873	0.436
a_0	50.2625	50.2625	51.2652	50.2688
b_0	13.1234	13.1234	13.52	13.228
n	0.6	0.600	0.6034	0.600
Quadratic error		1.e-12	0.0246	1 e-004
Iterations number		70	100	100
Computation time		85s	16.6 s	14.72 s

7. Experimental Identification

In order to appreciate the interest of the Park fractional model with the modal representation of the fractional integrator, we use this method to identify three possible models, using input/output data provided by an experiment at standstill operation (in this situation, $i_{ds}=i_{qs}$ and indeed $\omega = 0$).

7.1 Identification of the Conventional Model

Let i_{ds}^* be the measured current and $\hat{i}_{ds}$ be the simulated current, using conventional or fractional models.

$$J_c = \sum^{K} (i_{ds\,k}^* - \hat{i}_{ds\,k})^2 \qquad (45)$$

is the quadratic criterion which is minimized according to the output error technique (see [3],[5] and [18] for more details).The parameters with the conventional Park's model are defined by: $\underline{\theta}^T = [R_s \;\; R_r \;\; l_m \;\; l_r]$

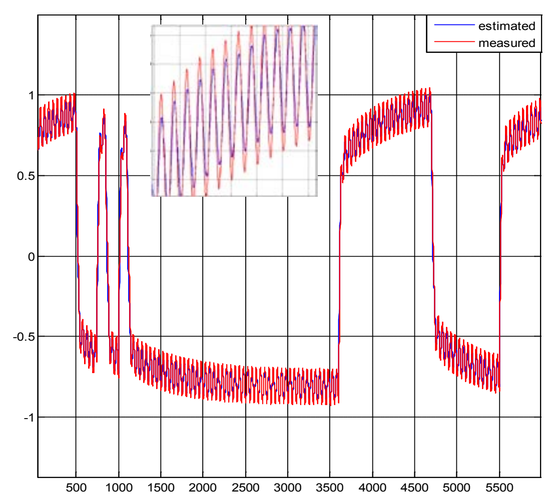

Fig. 9. Measured and estimated currents with conventional model

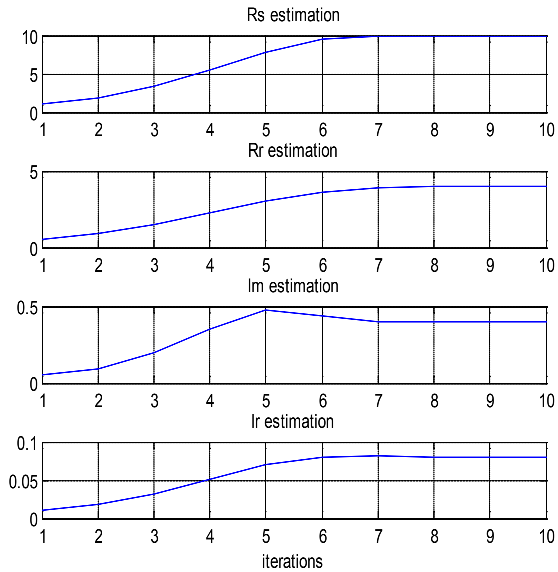

Fig. 10. Parameter estimation with conventional model

7.2 Identification of the Fractional Model H_n

The parameters with the fractional model H_n are defined by:

$$\underline{\theta}^T = [R_s \;\; l_m \;\; a_0 \;\; b_0 \;\; n]$$

As exhibited by figures 9 and 11, there is a good fit between measured and estimated currents with both conventional and fractional H_n models.

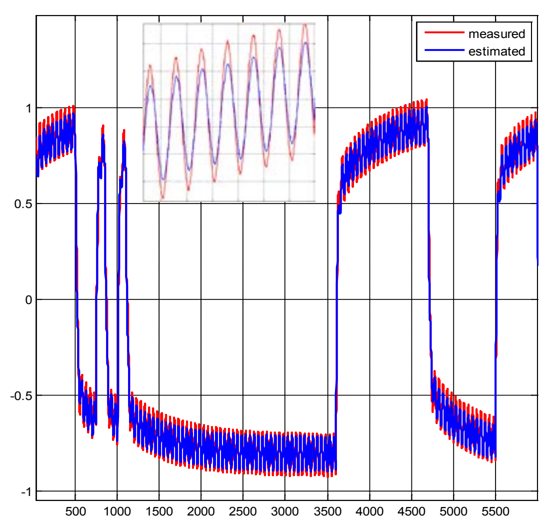

Fig. 11. Measured and estimated currents with fractional model $H_n(s)$

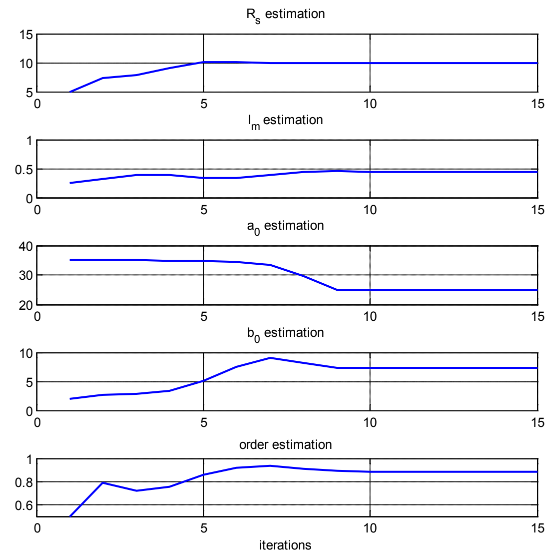

Fig. 12. Parameter estimation with fractional model $H_n(s)$

In order to appreciate the improvement of H_n model, it is necessary to compare the respective quadratic criterions (see table 2). It is obvious that the fractional model provides a better approximation of measurements than the conventional Park's model.

7.3 Identification of the Fractional Model $H_{n1,n2}$

The model (36) gives a good approximation only at low and medium frequencies [18]. In order to improve the fractional model (36) and particularly its high frequency approximation, a second model is proposed.

$$H_{n_1,n_2}(s) = \frac{b_0 + b_1 s^{n_1}}{a_0 + a_1 s^{n_1} + s^{n_1+n_2}} \tag{46}$$

It has been demonstrated (see for example [18]) that the phase of the fractional model has to be equal to $-\frac{\pi}{2}$ at high frequencies, i.e with a fractional order equal to 0.5.

If $s \to \infty$, $H_{n_1,n_2}(s) \to \frac{b_1 s^{n_1}}{s^{n_1+n_2}} = \frac{b_1}{s^{n_2}}$. Then if $n_2 = 0.5$, $H_{n_1,n_2}(s)$ model will provide a good approximation at high frequencies.

The parameters of the fractional model H_{n_1,n_2} are defined by:

$$\underline{\theta}^T = [R_s \;\; l_m \;\; a_0 \;\; a_1 \;\; b_0 \;\; b_1 \;\; n_1]$$

Because n_2 is set equal to 0.5, it is only necessary to estimate n_1.

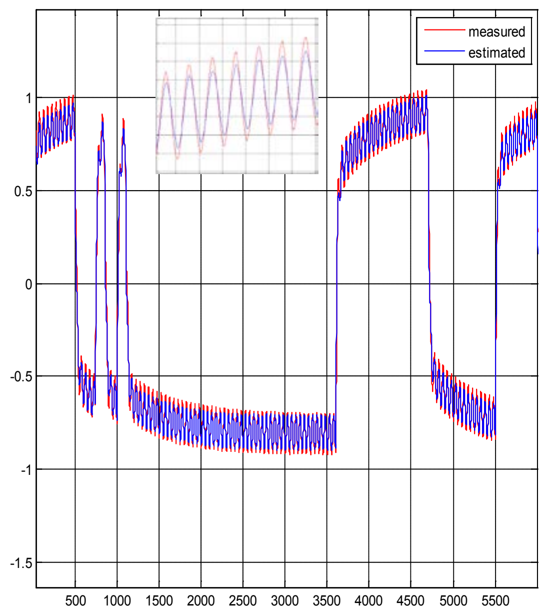

Fig.13. Measured and estimated currents with fractional model H_{n_1,n_2}

As previously, there is a good fit between measured and estimated currents demonstrated by figure 13. The figures 10, 12 and 14 represent the parameters variation during the identification.

The corresponding quadratic criterions of table 2 indicate that $H_{n_1,n_2}(s)$ performs a better approximation than the other models.

We present in the following table all the results of experimental parameter estimation.

Table 2. Estimated parameters

Classical model							
	R_s	R_r	l_m	l_r			
	9.99	4.01	0.397	0.0804			
Fractional model H_n							
	R_s	l_m	a_0	b_0	n		
	9.90	0.447	24.99	7.348	0.888		
Fractional model H_{n_1,n_2}							
	R_s	l_m	a_0	a_1	b_0	b_1	n_1
	9.98	0.385	21.07	-14.39	2.144	-0.601	0.103
Classical model : Quadratic criterion =11.47							
Fractional model H_n : Quadratic criterion = 9.80							
Fractional model H_{n_1,n_2} : Quadratic criterion =9.66							

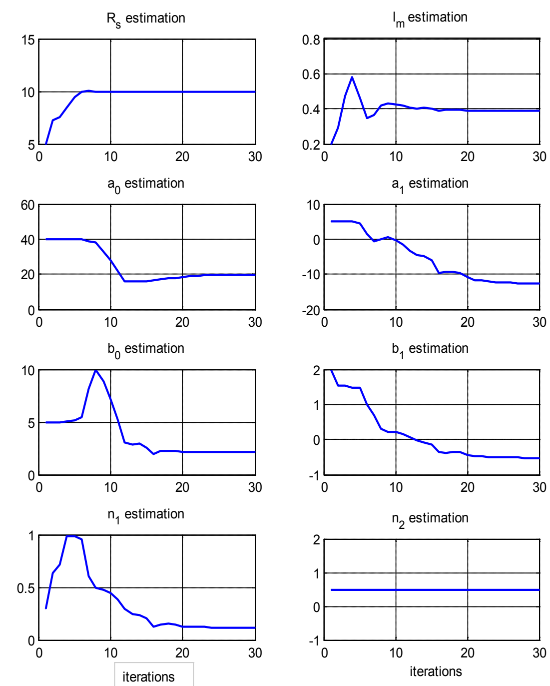

Fig. 14. Parameter estimation with fractional model H_{n_1,n_2}

8. Conclusion

In this paper, we have presented and compared some techniques and models for the identification of rotor skin effect in induction machines.

Thanks to Park's transformation we have obtained a conventional model in reference frame (dq) related to rotor. To take into account the diffusive phenomena of the skin effect, the Park's equivalent diagram with ladder model has been proposed. Then, we have replaced the ladder model by a fractional impedance.

The identification of the Park model with a fractional impedance has been performed by the output error method. Fundamentally, this method is based on the simulation of the model (and of sensitivity functions).

In a first step, we have demonstrated that the modal approach (for the simulation of the fractional integrator) performs the best compromise between precision, complexity and computation time.

In a second step, we have used this modal approach to compare three models with experimental data. The results

show clearly that the fractional models give better approximations than the conventional Park model.
Moreover, we have shown that a new fractional model with two derivatives is able to improve these experimental approximations.

Appendix

Using the complex formula, the inverse Laplace transform $L^{-1}(\frac{1}{s^n})$ is given by:

$$h(t) = \frac{1}{j2\pi}\int_{\gamma - j\infty}^{\gamma + j\infty} H(s)e^{st}ds \qquad \text{(A.1)}$$

We use the Bromwich contour shown in Fig. 9

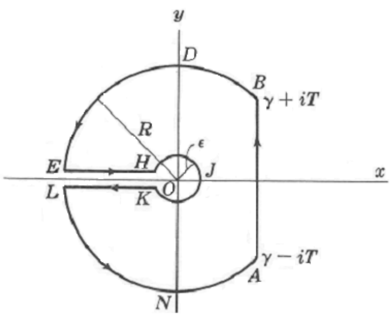

Fig.15. Bromwitch contour C

Thus, the impulse response $h(t)$ of any system can be calculated from its transfer function $H(s)$.

Because $H(s) = \frac{1}{s^n}$ $0 < n < 1$ is a multiform function, a cut is necessary in the complex plane, corresponding to the contour C of figure 2.
Thus we can write:

$$\frac{1}{j2\pi}\oint_C \frac{1}{s^n}e^{st}ds = \frac{1}{j2\pi}\left[\int_{AB} + \int_{BDE} + \int_{EH} + \int_{HJK} \int_{KL} + \int_{LNA}\right] \qquad \text{(A.2)}$$

Refering to Cauchy's theorem:

$$\frac{1}{j2\pi}\oint_C = 0 \qquad \text{(A.3)}$$

Because

$$\frac{1}{j2\pi}\left[\int_{BDE} + \int_{LNA}\right] = 0 \qquad \text{(A.4)}$$

and

$$\frac{1}{j2\pi}\int_{HJK} = 0 \qquad \text{(A.5)}$$

then :

$$\frac{1}{j2\pi}\int_{AB} = h_n(t) = \lim_{\substack{R\to\infty \\ \varepsilon\to 0}} -\frac{1}{j2\pi}\left[\int_{EH} + \int_{KL}\right] \qquad \text{(A.6)}$$

Finally we evaluate the integrals along the paths EH and KL.
Along EH,

$$s = xe^{j\pi} = -x$$
$$s^n = x^n e^{jn\pi} \qquad \text{(A.7)}$$
$$ds = -dx$$

and as s goes from $-\varepsilon$ to $-R$, x goes from ε to R.

$$\frac{1}{2\pi j}\int_{EH} = \frac{1}{2\pi j}\int_{-R}^{-\varepsilon}\frac{e^{st}}{s^n}ds = \frac{1}{2\pi j}\int_{\varepsilon}^{R}\frac{e^{-xt}}{(xe^{j\pi})^n}dx \qquad \text{(A.8)}$$

Along KL,

$$s = xe^{-j\pi} = -x$$
$$s^n = x^n e^{jn\pi} \qquad \text{(A.9)}$$
$$ds = -dx$$

$$\frac{1}{2\pi j}\int_{KL} = \frac{1}{2\pi j}\int_{-\varepsilon}^{-R}\frac{e^{st}}{s^n}ds = -\frac{1}{2\pi j}\int_{\varepsilon}^{R}\frac{e^{-xt}}{(xe^{-j\pi})^n}dx \qquad \text{(A.10)}$$

We thus obtain:

$$h(t) = \lim_{\substack{R\to 0 \\ \varepsilon\to 0}} \frac{1}{2\pi j}\left[\int_{\varepsilon}^{R}\frac{e^{-xt}}{(xe^{-j\pi})^n}dx - \int_{\varepsilon}^{R}\frac{e^{-xt}}{(xe^{j\pi})^n}dx\right] \qquad \text{(A.11)}$$

$$h(t) = \lim_{\substack{R\to 0 \\ \varepsilon\to 0}} \frac{1}{2\pi j}\int_{\varepsilon}^{R}\left(e^{jn\pi} - e^{-jn\pi}\right)x^{-n}e^{-xt}dx \qquad \text{(A.12)}$$

and finally:

$$h(t) = \int_0^{\infty}\frac{\sin n\pi}{\pi}x^{-n}e^{-xt}dx \qquad \text{(A.13)}$$

Because x corresponds to a frequency, let us define $w = x$. Notice that e^{-wt} is the impulse response $(z(w,t) = e^{-wt})$ of $\frac{1}{s+w}$ when its input is $v(t) = \delta(t)$.

Thus, in a more general situation, the response $z(w,t)$ of the elementary system to an input $v(t)$ verifies the differential equation:

$$\frac{\partial z(w,t)}{\partial t} = -w\, z(w,t) + v(t) \qquad \text{(A.14)}$$

and the output $x(t)$ of the fractional system is the weighted integral (with weight $\mu(w)$) of all the contributions $z(w,t)$ ranging from 0 to ∞:

$$y(t) = \int_0^\infty \mu(w) z(w,t) dw \quad (A.15)$$

$$\mu(w) = \frac{\sin(n\pi)}{\pi} w^{-n} \quad (A.16)$$

with $0 < n < 1$

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Abdelhamid JALLOUL was born in Monastir Tunisia on June 20, 1979. He received the engineer diploma and master degrees from National Engineering School of Sfax and M onastir, Tunisia, respectively in 2004 and 2006. He is currently working for the PhD degree at Tunis University, Tunisia. His research interests are in the fractional modeling of rotor skin effect in induction machines with application to control and diagnosis.

Khaled JELASSI was born in 1962, Tunisia. He received PhD in Electrical Engineering in 1991. He is currently Professor at Tunis University, Tunisia. His research interests are mainly in the area of modeling and di agnosis of the faults of the electromechanical systems.

Jean-Claude TRIGEASSOU was born in Libourne (33) France on december 12, 1946. He received the Ph.D. degree in automatic control from ENSM Nantes in 1980 and t he Doctorat d'Etat Es Sciences in automatic control from Poitiers University in 1987.
From 1988 to 2006, he has been pr ofessor at ESIP, an engineering school at Poitiers University. Since 2006, he is retired and Honorary Professor.
His major research interests have been in the method of moments with applications to identification and control and in the parameter estimation of continuous systems with application to the diagnosis of electrical machines.
At present, he his associated to the activities of the IMS-LAPS at Bordeaux University and his research works deal with modelling, stability, identification and control of fractional order systems.