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# Stability Study of Fuzzy Control Processes Application to a Nonlinear Second Order System

**Basma Zahra [1], Anis Sakly[2] and Mohamed Benrejeb[3]**
Ecole Nationale d'Ingénieurs de Tunis (ENIT), Laboratoire de Recherche en Automatique (LA.R.A)
**BP. 37, Le Belvédère, 1002, Tunis, Tunisia**

**Abstract**
In this paper, the stability study of fuzzy control systems is presented. The approach developed is based on the convergence of regular vector norms, where the comparison, the overvaluing principle and the Borne and Gentina criterion are used. The controller is of type PI-fuzzy with different partition of the two fuzzy inputs. The system to be controlled is a nonlinear system and the application example is of second order.

***Keywords:*** *Stability, Fuzzy, Mamdani, Controller, Nonlinear System, Second order***.**

## 1. Introduction

Fuzzy control has had great interest from the community of researchers over the last decades. This was due to the possibility of implementation without an exact mathematical model, sometimes even exact model is available, the use of fuzzy control seems reasonable and presents many advantages [2]. Several applications highlight the main advantages of fuzzy control [1][23][29][46][52]. Though, various kinds of fuzzy systems are widely used nowadays, this variety is issued from the types of fuzzy controllers in the closed loop of the system to be studied.

Stability analysis of such systems is still an open problem, in this way, many contributions were reported to discuss the cases of all types of fuzzy control systems [3][4][5][16][18][30][32][35][36][38][39][53][56][57][59][60][61], even though no general approach is available.

Sugeno in [48] classified fuzzy systems into three types. Type I, which was first introduced by Mamdani for steam-engine control [34], is characterised by the consequence part of rules given by a fuzzy set. This type is linguistically understandable since it uses fuzzy variables in both premises and consequence. The stability study of such systems hadn't the same opportunity as the other types [13][28][31]. Type II can be considered as a particular case of type I, the consequence is simply represented by a singleton [47][49]. For the last type of fuzzy systems which is called type III, the consequence is represented by a linear function. This type was first used for control of a model car [49][50]. The stability study of T-S fuzzy systems or systems of type III, was developed by many authors [16][20][47][48].

The concept of stability for nonlinear systems has been developed by many other authors [10][11][21][22][62], especially concerning the fuzzy logic controllers, since they are naturally nonlinear [6][12][19][35][37][51][55].

One of the authors presented a classification of the fuzzy system stability into two categories [28]: the nonlinear theory approach and the intuitive qualitative approach. The first class contains circle criterion, hyperstability approach, input-output theory, Lyapunov stability, phase plan criterion and other approaches. The second category is devoted to stability indices and energetic stability.

Lyapunov is one of the pioneers to study the stability of motion [27][33]. His approach is a general one with the fuzzy controller being modeled as Sugeno one. It is a sufficient condition for fuzzy control stability. If one of the subsystems is unstable, the closed loop fuzzy control system remains stable.

The phase plan criterion is a simple graphical approach. Inspecting the system trajectories will prove information on system stability or instability. This criterion is restricted to systems with an order less than two.

Concerning the circle criterion [26][40], it is restricted to a system that can be modeled as sector bound nonlinearity, it is also used for a time-variant system, in this case, the membership has to be symmetrical. The Popov approach, derived from the Kalman-Yakubovitch lemma for a time invariant system is a simple graphical method. The stability can be obtained by studying the Popov line. This method is restricted to systems with known control process.

There are some other approaches usually used for the stability of fuzzy control systems such that the passivity approach [14] and the hyperstability one [9][15][54]. We can also apply the notion of input-output stability to a Lur'e type system. In this case, the small gain theorem is frequently used [17].

Among the approaches used in the literature, a fuzzy controller is considered to be with a linear model to be controlled. In this paper, the fuzzy controller which is of type Mamdani corresponds to a nonlinear model.

In [39], Rambault proposed an extension of Popov criterion to a nonlinear model with two inputs, the error and its variation. The action surface is overvalued by a plan corresponding to a PD controller, where the input variation is a parameter. So, the nonlinearity becomes with only one input. The relative solution to stable models or with real poles, proposed in [12], uses state representation system and many of linear transformation on fuzzy controller inputs allowing the use of Popov criterion. Bühler proposed another algorithm based on Aizerman conjecture, in other way, the stability sector according to Popov coincides with that of Hurwitz [12]. By substituting the nonlinearity in the fuzzy controller by an equivalent linear characteristic, when the conjecture is checked, a criterion based on roots placement is proposed considering some important performances like the transient phenomenon damping.

Melin and Vidolov in [36] extended the approach in [40] to the case of two input controllers, the error and its variation, controlling stable or marginally stable systems. In the first time, for particular fuzzy controllers with two inputs [53], Vidolov established sufficient stability conditions returning the closed loop stable by using the Kalman-Yacubovitch theorem. Then, results are extended to a controller with four rules [61]. Finally, by considering a controller with strict partition of triangular fuzzy subsets such that at most four rules are activated simultaneously, results obtained remain valid.

In [42], there was an exploitation of the approach proposed in [6], [43] and [44] for stability study of continuous Mamdani fuzzy systems. This approach, was extrapolated to the case of nonlinear systems with the use of vector norms and comparison systems for particular case of input fuzzy subset partitions. In this paper, we present the stability study of fuzzy control systems for nonlinear systems to be controlled and for different input partition. So the following section is devoted to the description of the fuzzy control system containing a PI-fuzzy controller and the system to be controlled. The remaining parts deal with the determination of stability conditions of the fuzzy system by using Borne and Gentina criterion and vector norms approach. The stability conditions and the application example are applied for second order nonlinear process in the next sections.

## 2. System description

The PI-fuzzy control system considered in this study has two inputs the error $e$ and its derivative $de$ and one output the control derivative as shown in Fig. 1, where $k_e, k_{de}$ et $k_{du}$ are scale factors.

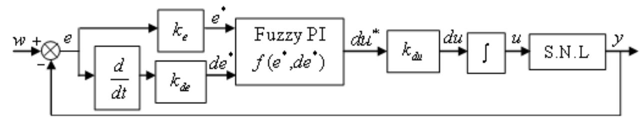


Fig. 1 Fuzzy Control system.

A particular class of Mamdani PI-fuzzy controllers is obtained by considering a strong triangular partition of the normalized variables $e^*$, $de^*$ and $du^*$ presented in Fig. 2 :

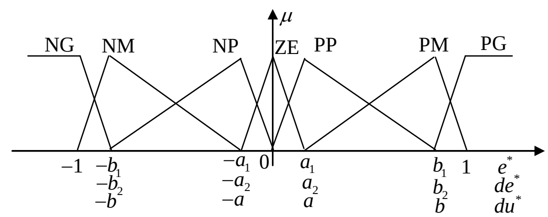


Fig. 2 Fuzzy Subset partition.

The rule base considered is an r×r traditional rule table that is of antidiagonal type such that the Mac Vicar-Whelan one (Table 1) [58].

Table 1: The Mac Vecar-Whelan table rule

| $e^*/de^*$ | NG | NM | NP | ZE | PP | PM | PG |
|---|---|---|---|---|---|---|---|
| NG | NG | NG | NG | NG | NM | NP | ZE |
| NM | NG | NG | NM | NM | NP | ZE | PP |
| NP | NG | NM | NP | NP | ZE | PP | PM |
| ZE | NG | NM | NP | ZE | PP | PM | PG |
| PP | NM | NP | ZE | PP | PP | PM | PG |
| PM | NP | ZE | PP | PM | PM | PG | PG |
| PG | ZE | PP | PM | PG | PG | PG | PG |

Let $\sigma(e^*, de^*)$ the surface in the space $(e^*, de^*, du^*)$, verifying the two properties [45] :

$$\begin{cases} \text{i) If } \sigma = 0 \text{ then the input-output characteristic surface } du^*(e^*, de^*) = 0 \\ \text{ii) It exists } k>0 \text{ such as } du^*(k\sigma - du^*) \geq 0 \text{ for all } e^* \text{ and } de^* \end{cases} \tag{1}$$

The first property means that the intersection of the overvaluing surface $\sigma$ with the plan $(e^*, de^*)$ is a part of the intersection of the characteristic surface $du^*(e^*, de^*)$ with the same plan. The curve $\sigma = 0$ is a straight line when the fuzzy input partition is identical for the inputs and it represents the second bisector. In general ( $a_1 \neq a_2$ and $b_1 \neq b_2$ ) the curve is piecewise linear as shown in Fig. 3.

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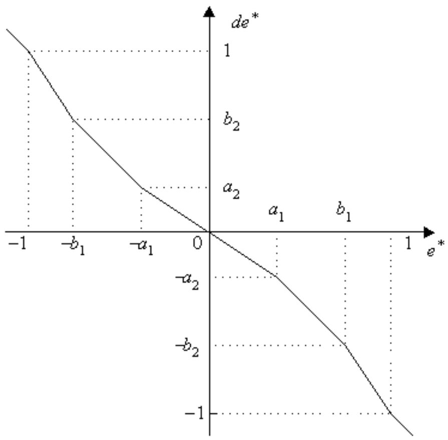


Fig. 3 Curve $\sigma = 0$.

For this case of input partition, the global stability is difficult to prove due to the nature of the curve $\sigma = 0$.
In consequence, we will study the local stability of the fuzzy control system in the neighborhood of the equilibrium point. In this zone, the part of the curve $\sigma = 0$ is segment, in these conditions the characteristic surface of the fuzzy controller can be locally overvalued by a plan.

## 3. Overvaluation of the fuzzy controller characteristic surface

The characteristic surface of the fuzzy controller $du^*(e^*, de^*)$ is locally overvalued and undervalued by two plans crossing the plan $(e^*, de^*)$ in a straight line. A part of this straight line is common with the curve $\sigma = 0$ in $e^* \in [-a_1, a_1]$ and $de^* \in [-a_2, a_2]$ such that :

$$\frac{de^*}{a_2} + \frac{e^*}{a_1} = 0 \tag{2}$$

The slopes of these plans are respectively $k_{\max}$ and $k_{\min}$, in this way the output is as follows:

$k_{\min}\left(\frac{de^*}{a_2} + \frac{e^*}{a_1}\right) \le du^* \le k_{\max}\left(\frac{de^*}{a_2} + \frac{e^*}{a_1}\right)$, for each point $(e^*, de^*)$ of $[-a_1, a_1] \times [-a_2, a_2]$, so we get :

$du^* = f(.)\left(\frac{de^*}{a_2} + \frac{e^*}{a_1}\right)$ then :

$$k_{\min} \le f(.) \le k_{\max} \tag{3}$$

where $f(.)$ is a nonlinear gain.
The values of $k_{\min}$ and $k_{\max}$ are determined numerically.

The fuzzy control system obtained is then given by Fig. 4.

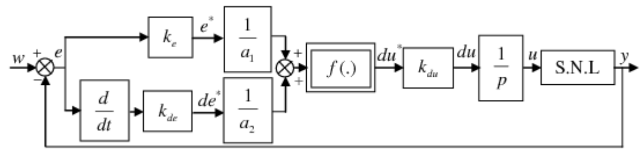


Fig. 4 Fuzzy control system for different fuzzy subset partition.

## 4. Proposed stability conditions

### 4.1 Problem formulation

The system to be controlled which is nonlinear is represented by the following state matrix given in the Frobenius form such that:

$$\begin{cases} \dot{x} = A(.)x + B(.)u \\ y = C(.)x \end{cases} \quad x \in \mathbb{R}^n \tag{4.a}$$

where:

$$A(.) = \begin{bmatrix} 0 & \dots & 0 & -a_n(.) \\ 1 & \ddots & \vdots & \vdots \\ \vdots & \ddots & 0 & \vdots \\ 0 & \dots & 1 & -a_1(.) \end{bmatrix}, B(.) = \begin{bmatrix} b_n(.) \\ \vdots \\ \vdots \\ b_1(.) \end{bmatrix}, C^T(.) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{4.b}$$

According to the diagram given in Fig. 4, we have $du = \dot{u}$, supposing that $v = du = \dot{u}$ and $\xi = u$, which leads to : $\dot{\xi} = v$.
The nonlinear system equipped with the integration can be represented by the following state matrix:

$$\dot{z} = \begin{bmatrix} A(.) & B(.) \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} v \quad z \in \mathbb{R}^{n+1} \tag{5}$$

where: $z = \begin{bmatrix} x \\ \xi \end{bmatrix}$ and $\dot{z} = \begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix}$

now, we suppose that:

$$A'(.) = \begin{bmatrix} A(.) & B(.) \\ 0 & 0 \end{bmatrix} \text{ and } B'(.) = B' = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}.$$

So, we can write $y = x_n = z_n$ where $x_n$ (respectively $z_n$) is the $n^{th}$ state variable of state vector $x$ (respectively $z$), thus : $y = C'(.)z$ where $C'(.) = C' = \begin{bmatrix} 0 & \dots & 1 & 0 \end{bmatrix}$, and we get :

$$v = k_{du} f(.)\left[\frac{de^*}{a_2} + \frac{e^*}{a_1}\right] = k_{du} f(.)\left[\frac{k_{de}\dot{e}}{a_2} + \frac{k_e e}{a_1}\right] \tag{6}$$

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After considering the system in the autonomous regime $(w=0)$, we get $(e=-y)$ and :

$$v = -k_{du} f(.)\left[\frac{k_{de}}{a_2}\dot{y} + \frac{k_e}{a_1} y\right]$$
$$= -k_{du} f(.)\left[\frac{k_{de}}{a_2} C'(.)\dot{z}'' + \frac{k_e}{a_1} C'(.)z''\right]$$

The closed loop of the fuzzy control system is given by:

$$\begin{aligned}\dot{z} &= A'(.)z + B'(.)v \\ &= A'(.)z - B'(.)k_{du} f(.)\left[\frac{k_{de}}{a_2}C'(.)\dot{z} + \frac{k_e}{a_1}C'(.)z\right] \\ &= A'(.)z - k_{du} f(.)\left[\frac{k_{de}}{a_2}B'(.)C'(.)\dot{z} + \frac{k_e}{a_1}B'(.)C'(.)z\right]\end{aligned} \quad (7)$$

which allows to write:

$$\left[I_{n+1} + k_{du}\frac{k_{de}}{a_2} f(.)B'C'\right]\dot{z} = \left[A'(.) - k_{du}\frac{k_e}{a_1} f(.)B'C'\right]z \quad (8)$$

supposing now: $\begin{cases} N = I_{n+1} + k_{du}\dfrac{k_{de}}{a_2} f(.)B'C' \\ M = A'(.) - k_{du}\dfrac{k_e}{a_1} f(.)B'C' \end{cases}$

where:

$$N = \begin{bmatrix} 1 & 0 & \dots & & \dots & 0 \\ 0 & 1 & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & & \ddots & \vdots \\ \vdots & & 0 & & \ddots & 0 \\ 0 & \dots & 0 & & k_{du}\dfrac{k_{de}}{a_2} f(.) & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 0 & \dots & 0 & -a_n(.) & b_n(.) \\ 1 & \ddots & & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & & 1 & -a_1(.) & b_1(.) \\ 0 & \dots & 0 & -k_{du}\dfrac{k_e}{a_1} f(.) & 0 \end{bmatrix}$$

and : $\det(N) = 1$

Finally, we obtain the following description of the closed loop system :

$$\dot{z} = A_C(.)z \quad (9)$$

where :

$$A_C(.) = N^{-1}M$$
$$= \begin{bmatrix} 0 & \dots & \dots & -a_n(.) & b_n(.) \\ 1 & & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & & 1 & -a_1(.) & b_1(.) \\ 0 & \dots & -k_{du}\dfrac{k_{de}}{a_2} f(.) & k_{du} f(.)\left(\dfrac{k_{de}}{a_2}a_1(.) - \dfrac{k_e}{a_1}\right) & -k_{du}\dfrac{k_{de}}{a_2} f(.)b_1(.) \end{bmatrix} \quad (10)$$

Now we have to make a basic change leading to a new representation of the system for establishing stability conditions.

We note : $z' = Pz$ and $z = P^{-1}z'$

where $P$ is a passage matrix such that :

$$P = \begin{bmatrix} 1 & \alpha_1 & \dots & (\alpha_1)^{n-1} & 0 \\ \vdots & \dots & & \vdots & \vdots \\ 1 & \alpha_{n-1} & \dots & (\alpha_{n-1})^{n-1} & \vdots \\ 0 & \dots & & 1 & 0 \\ 0 & \dots & \dots & 0 & 1 \end{bmatrix}$$

and then we get:

$$\begin{aligned}\dot{z}' &= PA_C(.)P^{-1}z' \\ &= A'_C(.)z'\end{aligned} \quad (11)$$

where:

$$A'_C(.) = \begin{bmatrix} \alpha_1 & 0 & \dots & \delta_1(.) & \upsilon_1(.) \\ \vdots & \ddots & & \vdots & \vdots \\ 0 & \dots & \alpha_{n-1} & \delta_{n-1}(.) & \upsilon_{n-1}(.) \\ \beta_1 & \dots & \beta_{n-1} & \sigma(.) & b_1(.) \\ \lambda_1(.) & \dots & \lambda_{n-1}(.) & \varphi(.) & \psi(.) \end{bmatrix} \quad (12)$$

and:

$$\beta_i = \frac{1}{\prod_{\substack{j=1 \\ j\neq i}}^{n}(\alpha_i - \alpha_j)} \qquad \forall i = 1,\dots,n-1 \quad (13)$$

$$\begin{cases} \delta_i(.) = -D(.,\alpha_i) & \forall i = 1,\dots,n-1 \\ D(.,\alpha) = \alpha^n + \sum_{j=1}^{n} a_j(.)\alpha^{n-j} \end{cases} \quad (14)$$

$$\begin{cases} \upsilon_i(.) = N(.,\alpha_i) & \forall i = 1,\dots,n-1 \\ N(.,\alpha) = \sum_{j=1}^{n} b_j(.)\alpha^{n-j} \end{cases} \quad (15)$$

$$\sigma(.) = -a_1(.) - \sum_{j=1}^{n-1}\alpha_j \quad (16)$$

$$\lambda_i(.) = -k_{du}\frac{k_{de}}{a_2} f(.)\beta_i \qquad \forall i = 1,\dots,n-1 \quad (17)$$

$$\psi(.) = -k_{du}\frac{k_{de}}{a_2} f(.)b_1(.) \quad (18)$$

$$\varphi(.) = k_{du}\frac{k_{de}}{a_2} f(.)\left[a_1(.) + \sum_{j=1}^{n-1}\alpha_j\right] - k_{du}\frac{k_e}{a_1} f(.) \quad (19)$$

To make the matrix $A'_C(.)$ in a simple arrow form, we consider a comparison system relative to the following regular vector norm $p$ :

$$p(z') = \left[|z'_1|,\dots,|z'_{n-1}|, \max\left\{|z'_n|,|z'_{n+1}|\right\}\right]^T \quad (20)$$

Let $Z = p(z')$ we define the overvaluing system relative to $p$ such that :

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$$\dot{Z} = M_C(.)Z \tag{21}$$

The matrix $M_C(.)$ is as follows :

$$M_C(.) = \begin{bmatrix} \alpha_1 & 0 & \dots & \gamma_1(.) \\ 0 & \ddots & & \vdots \\ \vdots & \dots & \alpha_{n-1} & \gamma_{n-1}(.) \\ \mu_1(.) & & \mu_{n-1}(.) & \mu(.) \end{bmatrix} \tag{22}$$

where :

$$\begin{aligned} \gamma_i(.) &= |\delta_i(.)| + |\upsilon_i(.)| \qquad \forall i = 1,...,n-1 \\ &= \left|\alpha_i^n + \sum_{j=1}^{n} a_j(.)\alpha_i^{n-j}\right| + \left|\sum_{j=1}^{n} b_j(.)\alpha_i^{n-j}\right| \end{aligned} \tag{23}$$

$$\begin{aligned} \mu_i(.) &= \max\{|\beta_i|, |\lambda_i(.)|\} \\ &= |\beta_i| \max\left\{1, k_{du}\frac{k_{de}}{a_2} f(.)\right\}, \forall i = 1,...,n-1 \end{aligned} \tag{24}$$

$$\begin{aligned} \mu(.) &= \max\{\sigma(.) + |b_1(.)|, |\varphi(.)| + \psi(.)\} \\ &= \max\left\{ \begin{aligned} & (-a_1(.) - \sum_{j=1}^{n-1}\alpha_j) + |b_1(.)|, \\ & \left| k_{du}\frac{k_{de}}{a_2} f(.)\left[a_1(.) + \sum_{j=1}^{n-1}\alpha_j\right] - k_{du}\frac{k_e}{a_1} f(.)\right| - k_{du}\frac{k_{de}}{a_2} f(.) b_1(.) \end{aligned} \right\} \end{aligned} \tag{25}$$

The matrix $M_C(.)$ is in a simple arrow form due to a reduction of the system order using the vector norm $p$.

Now we suppose that :

$$\mu_i(.) = |\beta_i| \text{ pour } k_{du}\frac{k_{de}}{a_2} f(.) < 1 \tag{26}$$

to apply the Borne and Gentina criterion on the matrix $M_C(.)$, in other way to get the nonlinear elements in the last line, constant $\mu_i(.)$.

The matrix $M_C(.)$ will be then in the following form :

$$M_C(.) = \begin{bmatrix} \alpha_1 & 0 & \dots & \gamma_1(.) \\ 0 & \ddots & & \vdots \\ & & \alpha_{n-1} & \gamma_{n-1}(.) \\ |\beta_1| & \dots & |\beta_{n-1}| & \mu(.) \end{bmatrix} \tag{27}$$

The nonlinear elements are situated in the last column and then we can state the following theorem [43] and [44].

### 4.2 Stability conditions

Theorem :

If there exist $\alpha_i<0$ for $i=1,...,n-1$, $\alpha_i \neq \alpha_j$ $\forall i \neq j$ such that $\forall Z \in S$ where $S$ is a neighbourhood domain of the equilibrium point :

$$\begin{aligned} & i)\, k_{du}\frac{k_{de}}{a_2} f(.) < 1 \\ & ii) - \mu(.) + \sum_{i=1}^{n-1}\gamma_i(.)\alpha_i^{-1}|\beta_i| > 0 \end{aligned} \tag{28}$$

then the equilibrium point $Z=0$ for the closed loop system is locally asymptotically stable.

## 5. Application

To illustrate the results developed and the stability conditions obtained, we discuss in this section the asymptotic stability of a fuzzy control system where the controller is of type PI-fuzzy described in the second section. The fuzzy control system is given in Fig. 4, and the system to be controlled is a nonlinear second order system given as follows :

$$\begin{cases} \dot{x} = A(.)x + B(.)u \\ y = C(.)x \end{cases} \tag{29.a}$$

where :

$$A(.) = \begin{bmatrix} 0 & -0.8 \\ 1 & -2 \end{bmatrix}, B(.) = \begin{bmatrix} b_1(.) \\ b_2(.) \end{bmatrix}, C^T(.) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{29.b}$$

For $v = du = \dot{u}$ and $\xi = u$, the description of the nonlinear system with the integration will be :

$$\dot{z} = \begin{bmatrix} A(.) & B(.) \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v \quad z \in \mathbb{R}^3, \text{ with } z = \begin{bmatrix} x \\ \xi \end{bmatrix} \tag{30}$$

let :

$$\begin{aligned} A'(.) &= \begin{bmatrix} A(.) & B(.) \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -0.8 & b_2(.) \\ 1 & -2 & b_1(.) \\ 0 & 0 & 0 \end{bmatrix}, \\ B'(.) &= B' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C'(.) = C' = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{aligned} \tag{31}$$

so, we get:

$$\begin{cases} \dot{z} = A'(.)z + B'v \\ y = C'z \end{cases} \tag{32}$$

When considering the local overvaluation of the fuzzy controller characteristic surface in the case of different fuzzy partition of the inputs, we can write:

$$du = v = k_{du} f(.)\left[\frac{k_{de}\dot{e}}{a_2} + \frac{k_e e}{a_1}\right] \tag{33}$$

and the system in the autonomous regime $(w = 0)$, we have :

$$v = -k_{du} f(.)\left[\frac{k_{de}}{a_2}C'\dot{z} + \frac{k_e}{a_1}C'z\right] \quad (34)$$

since $(y = C'z)$.

By replacing $v$ in (33) with its expression in (34) we get :

$$\dot{z} = A'(.)z - f(.)k_{du}\left[\frac{k_{de}}{a_2}B'C'\dot{z} + \frac{k_e}{a_1}B'C'z\right]$$

which leads to:

$$\dot{z}\left[I_3 + f(.)k_{du}\frac{k_{de}}{a_2}B'C'\right] = \left[A'(.) - f(.)k_{du}\frac{k_e}{a_1}B'C'\right]z \quad (35)$$

when supposing : $\begin{cases} N = I_3 + f(.)k_{du}\dfrac{k_{de}}{a_2}B'C' \\ M = A'(.) - f(.)k_{du}\dfrac{k_e}{a_1}B'C' \end{cases}$

we get :

$$N\dot{z} = Mz \quad (36)$$

where:

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & f(.)k_{du}\dfrac{k_{de}}{a_2} & 1 \end{bmatrix},\ M = \begin{bmatrix} 0 & -0.8 & b_2(.) \\ 1 & -2 & b_1(.) \\ 0 & -f(.)k_{du}\dfrac{k_e}{a_1} & 0 \end{bmatrix}$$

The description of the whole system allows to write :

$$\dot{z} = A_C(.)z \quad (37)$$

with :

$$A_C(.) = \begin{bmatrix} 0 & -0.8 & b_2(.) \\ 1 & -2 & b_1(.) \\ -f(.)k_{du}\dfrac{k_{de}}{a_2} & f(.)k_{du}\left(2\dfrac{k_{de}}{a_2} - \dfrac{k_e}{a_1}\right) & -f(.)k_{du}\dfrac{k_{de}}{a_2}b_1(.) \end{bmatrix}$$

We consider now the following basic change:

$$z' = Pz \quad (38)$$

with :

$$P = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then we get:

$$\dot{z}' = PA_C(.)P^{-1}z' = A'_C(.)z' \quad (39)$$

where :

$$A'_C(.) = \begin{bmatrix} \alpha & \delta(.) & \upsilon(.) \\ 1 & \pi(.) & b_1(.) \\ \lambda(.) & \varphi(.) & \psi(.) \end{bmatrix} \quad (40)$$

and :

$$\delta(.) = -\left(\alpha^2 + 2\alpha + 0.8\right) \quad (41)$$

$$\upsilon(.) = b_2(.) + b_1(.)\alpha \quad (42)$$

$$\lambda(.) = -f(.)k_{du}\frac{k_{de}}{a_2} \quad (43)$$

$$\varphi(.) = f(.)k_{du}\frac{k_{de}}{a_2}(2+\alpha) - f(.)k_{du}\frac{k_e}{a_1} \quad (44)$$

$$\psi(.) = -f(.)k_{du}\frac{k_{de}}{a_2}b_1(.) \quad (45)$$

To isolate the nonlinear elements of the matrix in one range and apply the Borne and Gentina criterion, we consider the following comparison system :

$$\dot{Z} = M_C(.)Z \quad (46)$$

such that :

$Z = p(z')$ and $p(z') = \left[|z'_1|, max\left\{|z'_2|,|z'_3|\right\}\right]^T$

and:

$$M_C(.) = \begin{bmatrix} \alpha & \gamma(.) \\ \mu_1(.) & \mu(.) \end{bmatrix}$$

where:

$$\gamma(.) = |\delta(.)| + |\upsilon(.)| = |\alpha^2 + 2\alpha + +0.8| + |b_2(.) + b_1(.)\alpha|$$

$$\mu_1(.) = \max\left\{1, |\lambda(.)|\right\} = \max\left\{1, \left|f(.)k_{du}\frac{k_{de}}{a_2}\right|\right\}$$

$$\mu(.) = \max\left\{\pi(.) + |b_1(.)|, |\varphi(.)| + \psi(.)\right\}$$

$$= \max\begin{cases} -(\alpha+2) + |b_1(.)|, \\ \left|f(.)k_{du}\dfrac{k_{de}}{a_2}(\alpha+2) - f(.)k_{du}\dfrac{k_e}{a_1}\right| - b_1(.)f(.)k_{du}\dfrac{k_{de}}{a_2} \end{cases} \quad (47)$$

Hypothesis :

We suppose that: $f(.)k_{du}\dfrac{k_{de}}{a_2} < 1$, with $\lambda(.) = -f(.)k_{du}\dfrac{k_{de}}{a_2}$,

and so $|\lambda(.)| < 1$ and $\mu_1(.) = \max\left\{1, |\lambda(.)|\right\} = 1$

In this way, the matrix $M_C(.)$ is such that:

$$M_C(.) = \begin{bmatrix} \alpha & \gamma(.) \\ \mu_1(.) & \mu(.) \end{bmatrix}$$

By applying the Borne and Gentina criterion, we get the following stability condition:

$$\alpha\mu(.) - |\gamma(.)| > 0 \quad (48)$$

$\alpha$ is chosen negative.

$$\alpha \max\left\{\pi(.) + |b_1(.)|, |\varphi(.)| + \psi(.)\right\} - \left(|\delta(.)| + |\upsilon(.)|\right) > 0$$

This condition allows to get the gathering of the two following systems:

System 1 :

$$\begin{cases}|\varphi(.)|+\psi(.)\ge \pi(.)+|b_1(.)|\\ \alpha\left(|\varphi(.)|+\psi(.)\right)>|\delta(.)|+|\upsilon(.)|\end{cases}\qquad(49)$$

and

System 2 :

$$\begin{cases}\pi(.)+|b_1(.)|\ge |\varphi(.)|+\psi(.)\\ \alpha\left(\pi(.)+|b_1(.)|\right)>|\delta(.)|+|\upsilon(.)|\end{cases}\qquad(50)$$

The system 1 gives :

$$\begin{cases}\pi(.)+|b_1(.)|\ge |\varphi(.)|+\psi(.)\\ \alpha\left(\pi(.)+|b_1(.)|\right)>|\delta(.)|+|\upsilon(.)|\end{cases}\qquad(50)$$

$$\begin{cases}\left|f(.)k_{du}\dfrac{k_{de}}{a_2}(2+\alpha)-f(.)k_{du}\dfrac{k_e}{a_1}\right|-f(.)k_{du}\dfrac{k_{de}}{a_2}b_1(.)\ge-(2+\alpha)+|b_1(.)|\\ \text{and}\\ \alpha\left(\left|f(.)k_{du}\dfrac{k_{de}}{a_2}(2+\alpha)-f(.)k_{du}\dfrac{k_e}{a_1}\right|-f(.)k_{du}\dfrac{k_{de}}{a_2}b_1(.)\right)>|\delta(.)|+|\upsilon(.)|\end{cases}\qquad(51)$$

For $f(.)k_{du}>0$ :

$$\begin{cases}f(.)k_{du}\left(\left|\dfrac{k_{de}}{a_2}(2+\alpha)-\dfrac{k_e}{a_1}\right|-\dfrac{k_{de}}{a_2}b_1(.)\right)\ge|b_1(.)|-(2+\alpha)\\ \text{and}\\ f(.)k_{du}\left(\left|\dfrac{k_{de}}{a_2}(2+\alpha)-\dfrac{k_e}{a_1}\right|-\dfrac{k_{de}}{a_2}b_1(.)\right)<\alpha^{-1}\left(|\delta(.)|+|\upsilon(.)|\right)\end{cases}$$

which gives: $\left(\left|\dfrac{k_{de}}{a_2}(2+\alpha)-\dfrac{k_e}{a_1}\right|-\dfrac{k_{de}}{a_2}b_1(.)\right)<0$ and so:

$$\begin{cases}f(.)k_{du}\le\dfrac{|b_1(.)|-(2+\alpha)}{\left|\dfrac{k_{de}}{a_2}(2+\alpha)-\dfrac{k_e}{a_1}\right|-\dfrac{k_{de}}{a_2}b_1(.)}\\ \text{and}\\ f(.)k_{du}>\dfrac{\left(|\alpha^2+2\alpha+0.8|+|b_2(.)+b_1(.)\alpha|\right)}{\alpha\left(\left|\dfrac{k_{de}}{a_2}(2+\alpha)-\dfrac{k_e}{a_1}\right|-\dfrac{k_{de}}{a_2}b_1(.)\right)}\end{cases}$$

We suppose now that: $k_e=k_{de}=1$ and $\alpha=-0.5$, so we get:

$$\begin{cases}f(.)k_{du}\le\dfrac{|b_1(.)|-1.5}{\left|\dfrac{1.5}{a_2}-\dfrac{1}{a_1}\right|-\dfrac{b_1(.)}{a_2}}\\ \text{and}\\ f(.)k_{du}>\dfrac{\left(|b_2(.)-0.5b_1(.)|+0.05\right)}{\dfrac{0.5b_1(.)}{a_2}-0.5\left(\dfrac{1.5}{a_2}-\dfrac{1}{a_1}\right)}\end{cases}$$

For particular values of $a_1$ and $a_2$ such as: $a_1=0.33$ and $a_2=0.5$ with $b_1(.)>0$ we get the following conditions:

$$\begin{cases}f(.)k_{du}\le\dfrac{b_1(.)-1.5}{0.0303-2b_1(.)}\\ \text{and}\\ f(.)k_{du}>\dfrac{\left(|b_2(.)-0.5b_1(.)|+0.05\right)}{-0.01515+b_1(.)}\end{cases}$$

In these conditions the stability domain is given in the following schema (Fig. 5):

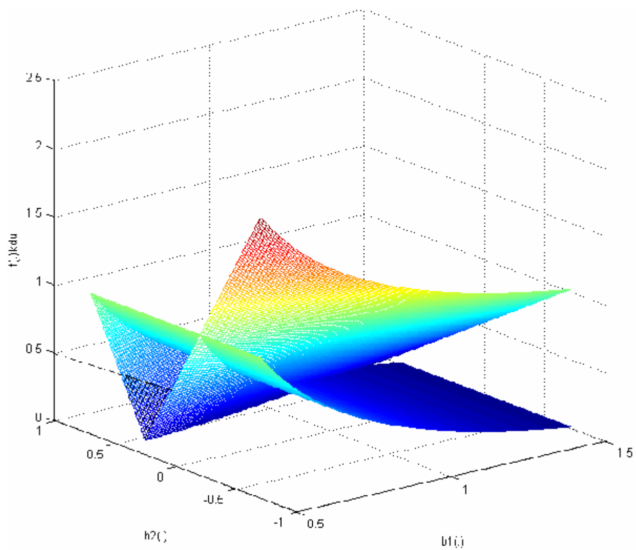


Fig. 5 Stability domain with report to $b_1(.)$ and $b_2(.)$ .

And the second system gives for $f(.)k_{du}>0$ :

$$\begin{cases}|b_1(.)|-(2+\alpha)\ge f(.)k_{du}\left(\left|\dfrac{k_{de}}{a_2}(2+\alpha)-\dfrac{k_e}{a_1}\right|-\dfrac{k_{de}}{a_2}b_1(.)\right)\\ \text{and}\\ |b_1(.)|-(2+\alpha)<\alpha^{-1}\left(|\alpha^2+2\alpha+0.8|+b_2(.)+\alpha b_1(.)\right)\end{cases}$$

So $|b_1(.)|-(2+\alpha)<0$, then for :

$k_e=k_{de}=1$, $\alpha=-0.5$ and $a_1=0.33$, $a_2=0.5$ with $b_1(.)>0$ we obtain the following stability conditions :

$$\begin{cases}f(.)k_{du}\left(0.03-2b_1(.)\right)\le b_1(.)-1.5\\ \text{and}\\ -0.1-2|b_2(.)-0.5b_1(.)|>b_1(.)-1.5\end{cases}$$

Which becomes for $b_1(.)>0.015$

$$\begin{cases} f(.)k_{du} \le \dfrac{b_1(.)-1.5}{0.03-2b_1(.)} \\ \text{and} \\ -2\left|b_2(.)-0.5b_1(.)\right| > b_1(.)-1.4 \end{cases}$$

Finally the stability domain for the whole fuzzy control system is the gathering of the two systems (system 1 and system 2).

## 6. Conclusion

The stability conditions of fuzzy control systems were presented in this paper. These conditions were deduced from stability study of overvaluing systems based on vector norms and the application of Borne and Gentina criterion. The controller is Mamdani PI-fuzzy one with a particular partition of the input subsets.
The nonlinear case of processes to be controlled was treated. Stability conditions for the fuzzy control system were obtained by applying Borne and Gentina criterion to the state matrix after making a basic change. For the nonlinear case, the basic change allowed to get a state matrix of order $(n+1)$ in thick arrow form. To return to the usual thin arrow form of the matrix and to get matrix with nonlinear elements isolated in only one range, we considered a comparison system relative to a regular vector norm. In this way, Borne and Gentina criterion was used to get sufficient stability conditions. These conditions were applied to a nonlinear second order system.

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**First Author** Basma Zahra, she received the Electrical Engineering diploma in 2003 from National engineering school of Sfax (ENIS), then the PhD degree in Electrical Engineering in 2011 from National engineering school of Tunis (ENIT). She is currently Assistant Professor at ISSAT Gafsa. Her research interests are in fuzzy systems, fuzzy control and stability of fuzzy control systems.

**Second Author** Anis Sakly, he received the Electrical Engineering diploma in 1994 from National engineering school of Monastir (ENIM), then the PhD degree in Electrical Engineering in 2005 from ENIT. His is currently Professor at ENIM. His research interests are in fuzzy control, particularly stability analysis and implementation of fuzzy control systems.

**Third Author** Mohamed Benrejeb, he obtained the diploma of “Ingénieur IDN” in 1973, the Master degree of Automatic Control in 1974, the PhD in Automatic Control from the university of Lille in 1976 and the DSc from the same university in 1980. Full Professor at ENIT since 1985. His research interests are in the area of analysis and synthesis of complex systems based on classical and non conventional approaches.