

# A Non-linear Controller for Single-Phase AC-AC Power Converter to meet UPS Performance Index

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## Abstract

This article focuses on AC-AC power converter that can be used for uninterruptible power supply (UPS). The converter is built on two stages: a AC-DC input stage and a DC-AC output stage. The two blocks are connected by an intermediate DC bus. The aim of control is threefold: i) power factor correction ii) regulation of DC bus iii) generating a sinusoidal voltage at the output. The synthesis of controllers has been achieved through the technique of nonlinear backstepping control. A detailed analysis of the stability control system is presented. The performances of regulators have been validated by numerical simulation in MATLAB / SIMULINK.

**Keywords:** UPS, AC-DC converter, DC-AC converter, power factor, Backstepping

## 1. Introduction

Single-phase UPS systems are widely used in low power applications such as computer systems, data storage, medical facilities and telecommunications. These systems are generally made by AC-AC converters which absorb currents with high levels of harmonic distortion. Several investigations have been published on the different topologies of AC-AC converters in order to develop control strategies to improve the energy behavior of these systems [1] [2] [3] [4].

Among these topologies, the one which is based on the cascade converters AC-DC and DC-AC presents a great feature: the simplicity of configuration, regardless of the values of voltage and frequency desired in the output of the converter. Figure 1 shows the block diagram of such a system. It consists of a half-bridge rectifier with a capacitive divider that converts the input AC line into a DC, and an inverter which is capable of generating from DC to AC with the desired amplitude and frequency.

Using the average model of AC-AC converters, several techniques for the design of controllers have been proposed, such as the classical regulators (PI or PID) using

small-signal linear models [5] [6]. In this article we will order our converter by the backstepping approach.

The idea is to calculate a control law to ensure that the derivative of a certain function (Lyapunov) defined positive is always negative. To do this, the system is decomposed into a set of sub nested systems. The calculation of Lyapunov function is, then, starting recursively from the inside of the loop. At each step, the order of the system is increased and the non stabilized part at the previous step is processed. The law of command to apply is synthesized at the last step. It must ensure, at all times, the overall stability of the system compensated while working in pursue or regulation [7] [8] [9].

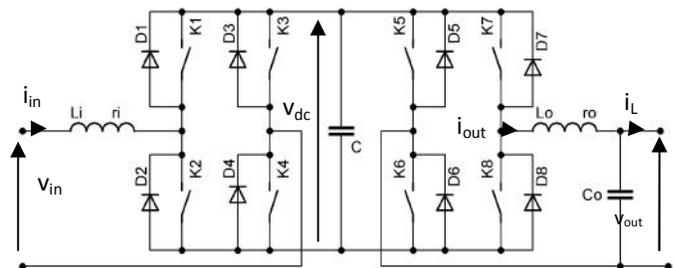


Fig . 1. AC-AC converter

## 2. Mathematical model of the converter

The converter shown in figure 1 consists of a rectifier and inverter each of which composed of an IGBT arm. These arms operate on the principle, widely known in the literature, pulse width modulation (PWM). Thus, arms (k<sub>1</sub>,k<sub>2</sub>,k<sub>3</sub>,k<sub>4</sub>) and (k<sub>5</sub>,k<sub>6</sub>,k<sub>7</sub>,k<sub>8</sub>) are associated respectively with the binary switching functions  $\tilde{s}_1$  et  $\tilde{s}_2$  such that:

$$\tilde{s}_1 = \begin{cases} 1 & \text{if } k_1 \text{ and } k_4 \text{ are ON, } k_2 \text{ and } k_3 \text{ are OFF} \\ -1 & \text{if } k_1 \text{ and } k_4 \text{ are OFF, } k_2 \text{ and } k_3 \text{ are ON} \end{cases}$$

$$\tilde{\sim}_2 = \begin{cases} 1 & \text{if } k_6 \text{ and } k_7 \text{ are ON, } k_5 \text{ and } k_8 \text{ are OFF} \\ -1 & \text{if } k_6 \text{ and } k_7 \text{ are OFF, } k_5 \text{ and } k_8 \text{ are ON} \end{cases}$$

The cyclical and binary signals  $\tilde{\sim}_1$  and  $\tilde{\sim}_2$  are the control inputs of the AC-AC. They vary from one period to another, and their variations can determine the trajectories of the state variables of the converter such as the currents in the inductors and the voltages across the capacitors.

The development of the model switched system is based on the application of Kirchhoff's laws. Thus, we obtain:

$$L_i \frac{di_{in}}{dt} = -r_i i_{in} + v_{in} - \tilde{\sim}_1 v_{dc} \quad (1a)$$

$$C \frac{dv_{dc}}{dt} = \tilde{\sim}_1 i_{in} - \tilde{\sim}_2 i_{out} \quad (1b)$$

$$L_o \frac{di_{out}}{dt} = -r_o i_{out} - v_{out} + \tilde{\sim}_2 v_{dc} \quad (1c)$$

$$C_o \frac{dv_{out}}{dt} = i_{out} - i_L \quad (1d)$$

The above model proves to be unsuitable for the development of continuous control laws since it involves, as input variables, the binary signals  $\tilde{\sim}_1$  and  $\tilde{\sim}_2$ . To overcome this inconvenience, the equivalent average model is used. Thus, we obtain the model:

$$L_i \dot{x}_1 = -r_i x_1 + v_{in} - u_1 x_2 \quad (2a)$$

$$C \dot{x}_2 = u_1 x_1 - u_2 x_3 \quad (2b)$$

$$L_o \dot{x}_3 = -r_o x_3 - x_4 + u_2 x_2 \quad (2c)$$

$$C_o \dot{x}_4 = x_3 - \bar{i}_L \quad (2d)$$

where  $x_1, x_2, x_3, x_4, u_1, u_2$  and  $\bar{i}_L$  represent, respectively, the means values, over a period of cutting, of variables  $i_{in}, v_{dc}, i_{out}, v_{out}, \tilde{\sim}_1, \tilde{\sim}_2$  and  $i_L$ .

Note that the mathematical model (2) is nonlinear because of the products involving the state variables and input signals. The non-standard form of this model leads us to choose techniques taking into account the nonlinearities such as approach backstepping.

### 3. Controller design

#### 3.1 The aim of controller

Our aim is to design a controller for the AC-AC converter that ensures both:

- Power factor correction (PFC) (grid side);
- the DC bus voltage regulation ;
- a sinusoidal voltage to the output of converter(load side)

The controller synthesis will be performed in three steps. First, an input current inner loop (regulator 1) is designed to cope with the PFC issue. In the second step, an output current inner loop (regulator 2) is designed to have a sinusoidal voltage whose amplitude and frequency are fixed by the reference signal  $x_4^* = v_{out}^* = V_{max} \sin(\hat{S}t)$ . In the end an outer voltage loop (regulator 3) is built-up to generate the reference signal, which will be used by the controller 1, to regulate the DC bus voltage at its desired reference value  $x_2^* = v_{dc}^*$ .

The proposed control system will have the structure shown in Figure 2.

Both controllers 1 and 2 will be synthesized by a technique using Backstepping approach and the third will be done by a simple proportional-integral corrector.

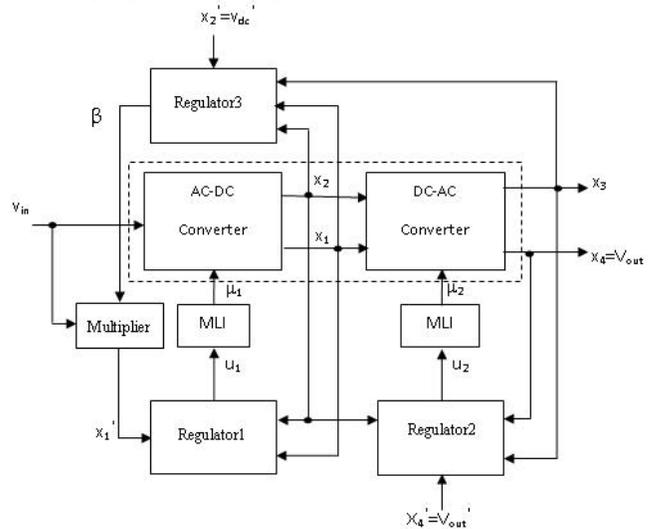


Figure 2. Basic structure of the controller

#### 3.2 Input current inner loop design (regulator 1)

The PFC objective means that the converter input current should be sinusoidal and in phase with the grid supply voltage. It amounts to ensuring current harmonics rejection. We therefore seek a regulator that enforces the current  $x_1$  to track a reference signal of the form  $x_1^* = S v_{in}$  when  $S \in \mathbb{R}^+$ . The block diagram in Figure 3 shows the basic structure of the control loop of the input current.

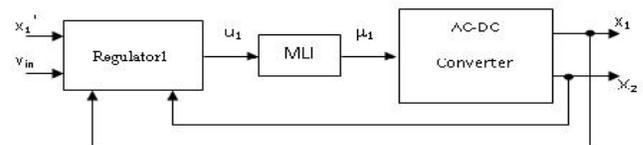


Figure 3. Structure of Input current regulator

Consider the tracking error  $z_1$  defined by:

$$z_1 = L_i(x_1 - x_1^*)$$

its dynamics is given by:

$$\dot{z}_1 = L_i(\dot{x}_1 - \dot{x}_1^*)$$

Let us use the Lyapunov candidate function:  $V_1 = 0.5z_1^2$

As its derivative with respect to time, is given by:

$$\dot{V}_1 = z_1 \dot{z}_1, \text{ the choice } \dot{z}_1 = -k_1 z_1$$

Where  $k_1$  is a positive constant synthesis, leads to a Lyapunov candidate function whose dynamics is negative definite.

So  $\dot{V}_1 = -k_1 z_1^2$ .

Therefore global asymptotic stability is achieved and  $z_1$  tends exponentially to 0.

Using (2a), (3) and (4), we will have

$$-k_1 z_1 = -r_i x_1 + v_{in} - u_1 x_2 - L_i \dot{x}_1^*$$

$x_2$  will initially be equal to  $\sqrt{2}E$  because the capacitor C of the DC bus is automatically loaded by the AC-DC converter that acts as a rectifier diode ( $D_1, D_2, D_3$  and  $D_4$ ) at system startup if we delay the switch control ( $k_1, k_2, k_3, k_4$ ).

Solving the previous equation with respect to  $u_1$  led to the backstepping control law as follows:

$$u_1 = \frac{1}{x_2} (k_1 z_1 - r_i x_1 + v_{in} - L_i \dot{x}_1^*) \quad (5)$$

### Proposition 1.

Considérons le convertisseur AC-AC de la figure 1 qui est décrit par le modèle moyen (2). Si la première dérivé de est disponible, alors la loi de commande (5) garantie la stabilité asymptotique globale du signal d'erreur  $z_1$ .

### 3.3 output current inner loop design (regulator 2)

The block diagram of figure 4 shows the control loop of the output voltage. It calculates the control law of the second stage of the power converter DC-AC to regulate the output voltage  $x_4$  to its desired reference value

$$x_4^* = V_{max} \sin(\check{S}t) .$$

The study will be developed for any grid connected to the output of the converter assuming that the current injected in the grid can be measured.

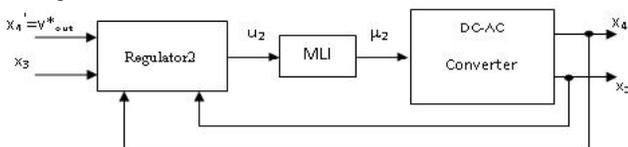


Figure 4. Output current inner loop

Taking into account the equations (2c) and (2d) the relative degree of the system, by respect to the variable  $x_4$ ,

is 2, the backstepping synthesis of the regulator will realized in two stages.

We define an error variable

$$z_4 = C_o(\dot{x}_4 - \dot{x}_4^*) \quad (7)$$

With (2d), the dynamics of the error  $z_4$  is given by:

$$\dot{z}_4 = x_3 - \frac{r_o}{L_o} z_4 - C_o \dot{x}_4^* \quad (8)$$

Consider the following candidate Lyapunov function:

$$V_4 = 0.5 z_4^2$$

its derivative with respect to time is given by:

$$\dot{V}_4 = \dot{z}_4 z_4$$

The choice  $\dot{z}_4 = -k_4 z_4$  (9)

Where  $k_4$  is a positive constant synthesis, leads to a Lyapunov candidate function whose dynamics is negative definite.

using (8) and (9) we will have

$$-k_4 z_4 = x_3 - \frac{r_o}{L_o} z_4 - C_o \dot{x}_4^* \quad (10)$$

if we choose  $x_3$  as virtual control input we deduce the stabilizing function:

$$x_3^* = -k_4 z_4 + \frac{r_o}{L_o} z_4 + C_o \dot{x}_4^* \quad (11)$$

As  $x_3$  is not the control input, a new error variable  $z_3$  between the virtual control  $x_3$  and its desired value  $x_3^*$  is introduced:

$$z_3 = x_3 - x_3^* \quad (12)$$

The dynamics of the error  $z_4$  is given by:

$$\dot{z}_3 = \dot{x}_3 - \dot{x}_3^* \quad (13)$$

Using (8), (9), (11) and (12), we will have :

$$\dot{z}_4 = z_3 - k_4 z_4 \quad (14a)$$

$$\dot{V}_4 = -k_4 z_4^2 + z_3 z_4 \quad (14b)$$

Now consider the dynamics of the error  $z_4$  taking into account (2c). so :

$$\dot{z}_3 = -\frac{r_o}{L_o} x_3 - \frac{1}{L_o} x_4 + \frac{1}{L_o} u_2 x_2 - \dot{x}_4^* \quad (15)$$

We see appear for the first time, the true signal of the control noted  $u_2$

The objective now is to stabilize the system ( $z_4, z_3$ ), for this we take as a candidate Lyapunov function the following function:

$$V_3 = \frac{1}{2} z_4^2 + \frac{1}{2} z_3^2$$

Its dynamics is given by

$$\dot{V}_3 = \dot{z}_4 z_4 + \dot{z}_3 z_3 .$$

Using (14a)  $\dot{V}_3 = -k_4 z_4^2 + z_3(z_4 + \dot{z}_3)$

The choice  $z_4 + \dot{z}_3 = -k_3 z_3$  (16)

Where  $k_3$  is a positive constant synthesis, which guarantees the negativity of the dynamics of the Lyapunov candidate function, because:

$$\dot{V}_3 = -k_4 z_4^2 - k_3 z_3^2 \leq 0$$

the equations (15) and (16) lead to:

$$-\frac{r_o}{L_o} x_3 - \frac{1}{L_o} x_4 + \frac{1}{L_o} u_2 x_2 - \dot{x}_3^* = -k_3 z_3 - z_4$$

We deduce then the following control law:

$$u_2 = \frac{L_o}{x_2} \left( \frac{r_o}{L_o} x_3 + \frac{1}{L_o} x_4 + \dot{x}_3^* - k_3 z_3 - z_4 \right) \quad (17)$$

**Proposition 2.**

Consider the AC-AC converter of Figure 1 which is described by the model means (2). The control law (17) guarantees the global asymptotic stability of error signals  $z_3$  and  $z_4$ . Moreover the dynamics of these errors is described by the following model:

$$\begin{pmatrix} \dot{z}_4 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} -k_4 & 1 \\ -1 & -k_3 \end{pmatrix} \begin{pmatrix} z_4 \\ z_3 \end{pmatrix}$$

**3.4 outer voltage loop design (regulator 3)**

The block diagram in Figure 5 shows the structure of the regulator of DC bus voltage.

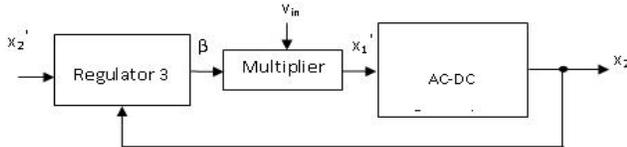


Figure 5. Structure for regulating the DC bus voltage

The aim of the outer loop is to generate a tuning law for the ratio  $\beta$  in such a way that the DC bus voltage  $x_2$  be regulated to a given reference value  $x_2^*$ . The first step in designing a loop is to establish the relation (model) between the ratio  $\beta$  (control input) and the DC voltage  $x_2$ .

**Hypothesis:**

**H1 :** The inner loop input current (regulator 1) and the inner loop output current (regulator 2) are supposed to have fast dynamics compared to of the outer loop of DC bus voltage (regulator 3).

**H2 :** The voltage drops across the different coils and their parasitic resistances are assumed negligible compared to the input and output voltages.

Based on the hypotheses H1 and H2, the control laws (5) and (17) reduce to:

$$u_1 = \frac{1}{x_2} v_{in} \quad (18a)$$

$$u_2 = \frac{1}{x_2} x_4 \quad (18b)$$

if we substitute  $u_1$  and  $u_2$  given by expressions (18) in (2b) the model becomes:

$$C\dot{x}_2 = \frac{1}{x_2} (v_{in}x_1 - x_4x_3)$$

This equation can be rewritten as follows:

$$\dot{x}_2 x_2 = \frac{1}{C} (v_{in}x_1 - x_4x_3). \quad (19)$$

If we set  $y = x_2^2$ , its dynamics becomes

$$\dot{y} = \frac{E^2}{C} S + p(t) \quad (20)$$

with  $p(t) = \frac{E^2}{C} S \cos(2\check{S}t) - \frac{2}{C} x_3 x_4$

Note that the model (20) can be seen as an integrator perturbed by the signal  $p(t)$ . The controller design is based on the average model as follows:

$$\dot{\bar{y}} = k\bar{S} + \bar{p}(t) \quad (21)$$

with  $k = \frac{E^2}{C}$   $\bar{S} = \frac{1}{T} \int_0^T S dt$

$\bar{p}(t) = -\frac{2}{CT} \int_0^T x_3 x_4 dt$   $\bar{y} = \frac{1}{T} \int_0^T y dt$

Where T is the period of the network.

The control model (21) has been verified by simulation. So, the step response results in a signal y (the square of the output voltage) ramp type perturbed by undulations of low amplitude.

The system (21) can be stabilized using a simple PI controller whose transfer function is given by

$$C(s) = k_p + k_i \frac{1}{s}$$

The block diagram of the controlled system is shown in Figure 6.

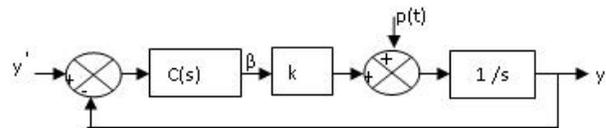


Figure 6. Equivalent loop voltage

In closed loop, the output signal y depends on the reference signal  $y^*$  and the disturbing signal  $p(t)$  by the equation:

$$Y(s) = F(s).Y^*(s) + G(s).P(s) \quad (22)$$

With:

$$F(s) = \frac{1 + \dagger_1 s}{1 + \frac{2\check{S}_0}{\check{S}_0} s + \frac{s^2}{\check{S}_0^2}} \quad G(s) = \frac{\dagger_2 s}{1 + \frac{2\check{S}_0}{\check{S}_0} s + \frac{s^2}{\check{S}_0^2}}$$

Where

$$\dagger_1 = k k_p / k_i, \quad \dagger_2 = 1 / k_i$$

$$\tilde{S}_0 = \sqrt{k_i} \quad , \quad \kappa = \frac{1}{2} \frac{k k_p}{\sqrt{k_i}}$$

The transfer relation (22) shows that the proposed controller guarantees a perfect pursuit  $\lim_{t \rightarrow \infty} (y^* - y) = 0$  and a rejection disturbance since  $G(s)$  contains a derivator effect.

The Bode diagram of Figure 7 shows the frequency response of the transfer function  $F(s)$ .

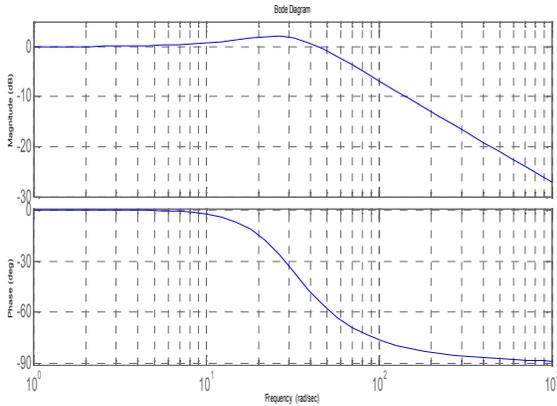


Figure 7. Bode diagram of  $F(s)$

#### 4. SIMULATION RESULTS

The performance of the proposed controller has been validated by simulation in the environment MATLAB / SIMULINK. Table 1 shows all the parameters of the controlled system.

Figures 8 to 14 shows the simulation results when reference of the DC bus voltage ( $v_{dc}$ ) is set at 700V and the output voltage ( $v_{out}$ ) is a sinusoidal signal with a RMS amplitude equal to 230V and 50Hz. Figure 8 shows that the input current of the converter is sinusoidal. In the Figure 9, we note that current and input voltage are sinusoidal and in phase. This shows that the power factor correction is perfectly realized. Figures 10 and 11 show the evolution of the output voltage delivered by the converter. In particular we note that it is the sinusoidal waveform with the desired characteristics (amplitude and frequency).

Figure 12 shows that the DC bus voltage follows perfectly (on average) its reference. Finally in Figures 13 and 14, are presented the control signals  $\tilde{u}_1$  and  $\tilde{u}_2$ , it is clear that they are bounded.

Item	value	
grid	E	$230\sqrt{2}$ V
	$\omega$	$100\pi$ rad/s
AC-DC converter	$L_i$	3.3mH

DC-AC converter	$r_i$	30m
	C	4700 $\mu$ F
	$L_o$	2mH
	$r_o$	30m
Load	$C_o$	2000 $\mu$ F
	$R_C$	10
Switching frequency	$L_C$	2mH
	$f_{PWM}$	10Khz

Table 1. simulation parameters

#### 5. CONCLUSION

In this paper, we presented a new control technique using the backstepping approach to control AC-AC converters used in the Uninterruptible Power Systems (UPS). The control objective is threefold i) correcting the power factor of the network side ii) generating a perfectly sinusoidal voltage to the inverter output iii) regulating the DC bus voltage between the rectifier stage and inverter stage AC-AC converter phase used. The studied system is described by a representation of nonlinear state average of order 5. The controller has been synthesized using advanced tools of nonlinear control such as stability in the sense of LYAPUNOV. It was shown by simulation that the proposed controller guarantees the desired tracking performance and stability.

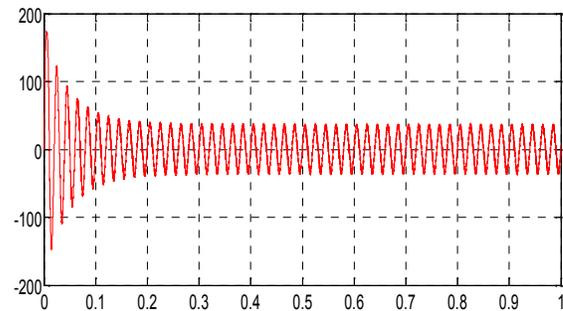


Figure 8.  $i_{in}$  current

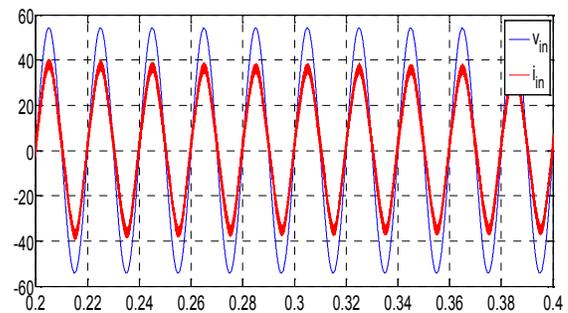


Figure 9.  $i_{in}$  current and  $v_{in}$  ( $v_{in}/6$ ) voltage

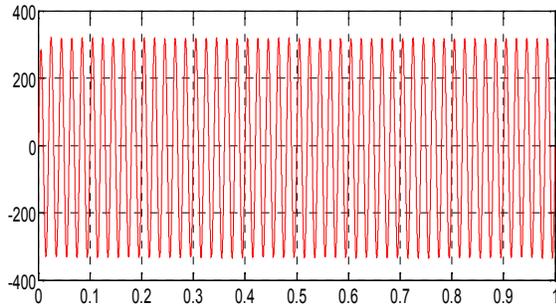


Figure 10. Output voltage  $v_{out}$

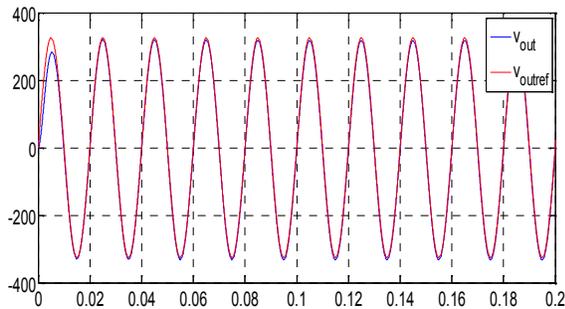


Figure 11. Output voltage of DC-AC converter and its reference

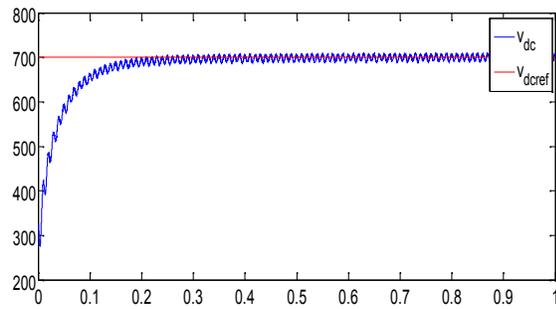


Figure 12. DC bus voltage  $v_{dc}$  and the reference

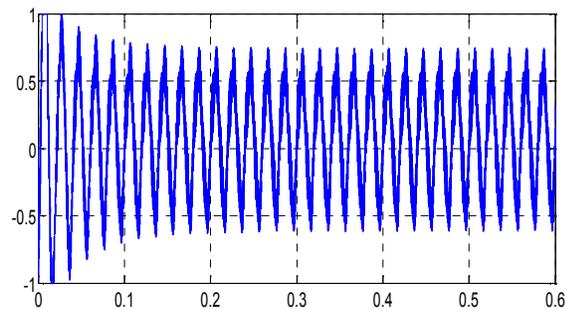


Figure 13. The control law  $\sim_1$

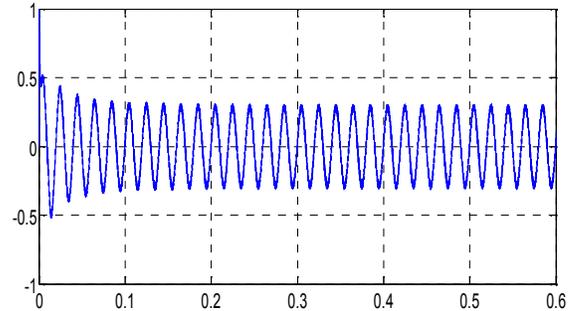


Figure 14. The control law  $\sim_2$

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