Control of Free-floating Space Robotic Manipulators base on Neural Network

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Abstract
For problems of trajectory tracking of the free-floating space robot model with uncertainties in task space, neural networks adaptive control strategy is put forward by this paper. Because the non-linear system model can not be obtained accurately, neural network is used to directly identify all parts of the system parameters through GL matrix and its multiplication operator “·”. Robust controller is designed to eliminate the approximation errors of neural network and external disturbances. The control strategy neither requires an estimate of inverse dynamic model, nor calculates the inverse Jacobin matrix. Global asymptotic stability based on Lyapunov theory is proved by the paper. Simulation results show that higher control precision is achieved. The control strategy has great value in engineering applications.

Keywords: Neural network, GL matrix, Space robot, Adaptive control, Global asymptotic stability

1. Introduction
With the development of space technology, space robot will play an important role in space exploration. Owing to the economization of fuel, increase life-span of space robot, and decrease in launch expenditure, positions and attitude of the base are entirely free. Thus there is intense dynamic coupling existing between the manipulator and base. Meanwhile, there are many uncertainties existing in the space robot dynamic model, for example, the dynamic model of manipulator mass, inertia matrix and load quality can not be accurately acquired, and external disturbance signals have a certain impact on the controller. To eliminate these non-linear factors, many advanced control strategies such as robust control [1~3], adaptive control [4]-[5], fuzzy control [6]-[11] and neural network control[12]-[15] have been used in space robot tracking control.
[16]-[17]bring forward adaptive control methods. In the process of designing, the parameters of dynamic equations need be linearized, so complicated pre-calculation is required. [18] proposes an fuzzy-neural control method which does not requires the exact model of robot. But much parameters is adjusted, that affects the real-time. [19] has presented a neural network control method, uncertain model can be identified adaptively by neural network, but this control scheme only can guarantee the system uniformly ultimately bounded (UUB).

For the above shortcomings, this paper presents a neural network adaptive control method. Considering that exact model is difficult to obtain, this control method use the neural network to identify system parameters. robust controller is designed to eliminate the approximation errors of neural network and external disturbances. The control method neither requires an estimate of inverse dynamic model, nor calculates the inverse Jacobian matrix. The amount of calculation is decreased by adopting GL matrix and its multiplication operator. Based on Lyapunov theory, global asymptotic stability of the closed-loop system is proved. Simulation results show that the controller can achieve higher precision.

2. Dynamic Equations of Space Robot in Task Space
Free-floating space robot dynamic equation can be written as follow[20]:

\[ M(q) \ddot{q} + B(q, \dot{q}) \dot{q} = \tau \]  

(1)
Where \( q, \dot{q}, \ddot{q} \in \mathbb{R}^n \) are joint position, velocity and acceleration vectors; \( M(q) \in \mathbb{R}^{n\times n} \) is symmetric positive definite inertia matrix; \( B(q, \dot{q}) \in \mathbb{R}^{n\times1} \) is Coriolis/centrifugal forces; \( \tau \in \mathbb{R}^{n\times1} \) is control torque.

As the robot in task is generally given by the cartesian coordinate system. The paper can select the displacement of the robot’s end-actuator in Cartesian space as the system output \( y \). Thus the system’s augmentation output can be written as:

\[
y = h(q) = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}
\]  

(2)

Which \( y \in \mathbb{R}^n \) indicates the positions and attitudes of the manipulator end-actuator in Cartesian coordinates; for planar two link robot, \( n = 2 \). When calculating the derivative of it. The paper can get the following equation:

\[
y' = J(q)q' = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}
\]  

(3)

In the aboving equation, \( J(q) = [J_\theta(q), \ J_\gamma(q)] \) represents the Jacobian matrix, \( J_\theta = \partial y / \partial q \), \( J_\gamma = \partial y / \partial \dot{q} \). The paper hypothesizes that \( J_\gamma \) is a non-singular matrix, and then \( J \) is reversible. Thus by the equation (3) and equation (1), taking the external disturbance \( d \) into account. The paper finally can obtain the dynamic equation of the space robot in task space:

\[
D\ddot{y} + C\dot{y} + d = \tau
\]  

(4)

Where \( C = J^TBJ^{-1} - J^TMJ^{-1}JJ^{-1}, \ D = J^TMJ^{-1}, \ \tau = J^T\tau_q \).

The dynamic equation (4) of space robot in task space has the following properties [21]:

**Property1:** \( D(q) \) is reversible and bounded.

**Property2:** For any \( Z \in \mathbb{R}^n \), there is \( \frac{1}{2} Z^T D = Z^T C Z \).

3. Design of Neural Network Controller

3.1 GL matrix and its Multiplication operators

When the symbol "\{\}" is defined as the GL matrix and "\(\cdot\)" as its method operator, the GL vector \( \{\Theta\} \) and its transpose \( \{\Theta\}^T \) may be defined as the following equation:

\[
\{\Theta\} = \begin{bmatrix} \theta_{11} \\ \theta_{12} \\ \vdots \\ \theta_{1n} \\ \theta_{21} \\ \theta_{22} \\ \vdots \\ \theta_{2n} \\ \vdots \\ \vdots \\ \theta_{n1} \\ \theta_{n2} \\ \vdots \\ \theta_{nn} \end{bmatrix} = \begin{bmatrix} \{\theta_1\} \\ \{\theta_2\} \\ \vdots \\ \{\theta_n\} \end{bmatrix}
\]

(5)

\[
\{\Theta\}^T = \begin{bmatrix} \theta_{11}^T \\ \theta_{12}^T \\ \vdots \\ \theta_{1n}^T \\ \theta_{21}^T \\ \theta_{22}^T \\ \vdots \\ \theta_{2n}^T \\ \vdots \\ \vdots \\ \theta_{n1}^T \\ \theta_{n2}^T \\ \vdots \\ \theta_{nn}^T \end{bmatrix}
\]

(6)

For a given GL matrix \( \{\Xi\} \)

\[
\{\Xi\} = \begin{bmatrix} \xi_{11} \\ \xi_{12} \\ \vdots \\ \xi_{1n} \\ \xi_{21} \\ \xi_{22} \\ \vdots \\ \xi_{2n} \\ \vdots \\ \vdots \\ \xi_{n1} \\ \xi_{n2} \\ \vdots \\ \xi_{nn} \end{bmatrix} = \begin{bmatrix} \{\xi_1\} \\ \{\xi_2\} \\ \vdots \\ \{\xi_n\} \end{bmatrix}
\]

(7)

Then the GL multiplication of \( \{\Theta\}^T \) and \( \{\Xi\} \) may be defined as a \( n \times n \) matrix, as follows:

\[
\{\Theta\}^T \{\Xi\} = \begin{bmatrix} \theta_{11}^T \xi_{11} + \theta_{12}^T \xi_{12} + \cdots + \theta_{1n}^T \xi_{1n} \\ \theta_{21}^T \xi_{21} + \theta_{22}^T \xi_{22} + \cdots + \theta_{2n}^T \xi_{2n} \\ \vdots \\ \theta_{n1}^T \xi_{n1} + \theta_{n2}^T \xi_{n2} + \cdots + \theta_{nn}^T \xi_{nn} \end{bmatrix}
\]

(8)

Where, it should be noted that: the GL multiplication should be calculated firstly in the mixed calculation of the matrix and the GL matrix multiplication operator.

3.2 Design of neural network adaptive controller

For the dynamic model (4) of space robot, the paper can define \( y_r \) as the reference trajectory, \( y_d \) as the ideal trajectory, \( e(t) \) as the position tracking error, \( r \) as the filtering error slip surface, and \( \Lambda \in \mathbb{R}^{m\times n} \) as the positive definite matrix, then:

\[
\dot{y}_r(t) = y_d(t) + \Lambda e(t)
\]

(9)

\[
e(t) = y_d(t) - y(t)
\]

(10)

\[
r(t) = \dot{e}(t) + \Lambda e(t)
\]

(11)

**Lemma**[22]. Let \( e(t) = h(t) \ast r(t) \), which \( \ast \) represents convolution, \( h(t) = L^\ast H(s) \) and \( H(s) \) is a \( n \times n \) class transfer function with strictly exponentially stable, if \( r \in L^2 \), then \( e \in L^2 \cap L^\infty \), \( \dot{e} \in L^2 \), \( e \) is continuous. When \( t \to \infty \), \( e \to 0 \), \( r \to 0 \), \( \dot{e} \to 0 \), the error
equation of closed-loop system of free-floating space robot can be written as:
\[ D\dot{r} + Cr = D\dot{y}_r + Cy_r + d - \tau \]  
(12)

If the robot modeling is accurate, and \( d = 0 \). The paper can design the following control law equation to guarantee the global asymptotic stability of closed-loop system.
\[ \tau = D\dot{y}_r + Cy_r + K_v r \]  
(13)

Where \( K_v \) represents positive definite matrix.

Proof: to take Lyapunov function as:
\[ V = \frac{1}{2} r^T Dr \]  
(14)

To calculate its differential, the paper can obtain the following equation:
\[ \dot{V} = r^T D\dot{r} + \frac{1}{2} r^T Dr \]  
(15)

To combine the closed-loop error equation (2) and the control law equation (13), the paper can get to the following equation:
\[ \dot{V} = -r^T K_v r \leq 0 \]  
(16)

However, in practical engineering, the free-floating space robot model \( D(q) \) and \( C(q, \dot{q}) \) are difficult to accurately obtain, and the external disturbance \( d \) exists in system, these nonlinear uncertainties will cause the control performance to degrade.

Considering that the neural network has good nonlinear approximation ability, the paper can adopt RBF local generalization network to approximate the uncertain parts \( D(q) \), \( C(q, \dot{q}) \) in the unknown system. Thus the learning speed can be accelerated greatly and local minimum problems can be avoided. Then the neural network model equation can be written as:
\[ D(q) = \sum_i \theta_{ij}^T \xi_{ij}(q) + \epsilon_{dj}(q) \]  
(17)

\[ C(q, \dot{q}) = \sum_i \alpha_{ij} \phi_{ij}(z) + \epsilon_{cij}(z) \]  
(18)

Where, \( z = (q, \dot{q}) \), \( \theta_{ij} \), \( \alpha_{ij} \) represents weights of the neural network, \( \xi_{ij}(q) \), \( \phi_{ij}(z) \) represents radial basis function of the input vector. And \( \epsilon_{dj}(q) \) and \( \epsilon_{cij}(z) \) respectively represents its modeling errors, and is assumed to be bounded.

When using GL matrix and its multiplication operator, \( D(q) \) and \( C(q, \dot{q}) \) can be written as:
\[ D(q) = \Theta^T [(\Xi(q))] + E_D(q) \]  
(19)
\[ C(q, \dot{q}) = [A]^T [(Z(z))] + E_C(z) \]  
(20)

Where, \( \Theta \), \( \Xi(q) \), \( A \) and \( Z(z) \) represent the GL matrices, and their elements respectively are \( \theta_{ij} \), \( \alpha_{ij} \), \( \xi_{ij}(q) \) and \( \phi_{ij}(z) \). \( E_D(q) \) and \( E_C(z) \) respectively represents the matrix of the modeling errors \( e_{dj}(q) \) and \( e_{cij}(z) \).

Where. The paper respectively define \( \hat{\Theta} \) and \( \hat{A} \) as the estimate value of \( \Theta \) and \( A \), and \( \hat{\Theta} \) and \( \hat{A} \) as their estimation errors.
\[ \hat{\Theta} = \Theta - \hat{\Theta} \]  
(21)
\[ \hat{A} = A - \hat{A} \]  
(22)

Then the controller equation (13) should be revised as:
\[ \tau = [[\hat{\Theta}^T \cdot (\Xi(q))] \dot{y}_r + [[\hat{A}^T \cdot (Z(z))] \dot{y}_r + K_v r + k_s sgn(r) \]  
(23)

Where \( k_s > ||E||, E = E_D \dot{y}_r + E_C \dot{y}_r + d \).

The adaptive law is designed as:
\[ \dot{\hat{\theta}}_i = \Gamma_i \cdot [\xi_i(q)] \dot{y}_r + i \]  
(24)
\[ \dot{\hat{\alpha}}_i = Q_i \cdot [\phi_i(z)] \dot{y}_r + i \]  
(25)

Where \( \Gamma_i = \Gamma_i^T > 0 \), \( Q_i = Q_i^T > 0 \), \( \hat{\theta}_i \) and \( \hat{\alpha}_i \) are the vectors of the matrices whose elements are respectively \( \hat{\theta}_j \) and \( \hat{\alpha}_j \).

To put the equation (17) and (18) into the equation (4), and combine into the equation (23). The paper can get to the following equation:
\[ \hat{\Theta}^T \cdot (\Xi(q)) \dot{y}_r + [[\hat{A}^T \cdot (Z(z))] \dot{y}_r + K_v r + k_s sgn(r) - d \]  
(26)

Putting \( \hat{y}_r = \dot{y}_r - r \), \( \hat{y}_r = \dot{y}_r - r \) into the equation (26), and reducing it, the paper can get to:
\[ \hat{\Theta}^T \cdot (\Xi(q)) \dot{y}_r + [[\hat{A}^T \cdot (Z(z))] \dot{y}_r + K_v r + k_s sgn(r) \]  
(27)
Putting the equation (4) into the aboving equation to calculate, and the paper can obtain the following equation:

\[ \dot{D}r + Cr + K_r r + k_s \text{sgn}(r) \]

\[ = \{\dot{\Theta}^T \cdot \{\Xi(q)\}\} \ddot{y}_r + \{\dot{A}^T \cdot \{Z(z)\}\} \dot{y}_r + E \quad (28) \]

3.3 Stability Analysis

The paper can define the following Lyapunov functions to prove the stability of closed-loop system.

Proof:

\[ V = \frac{1}{2} r^T Dr + \frac{1}{2} \sum_{i=1}^{n} \ddot{\theta}^T_i \Gamma_i^{-1} \ddot{\theta}_i + \frac{1}{2} \sum_{i=1}^{n} \dddot{\alpha}_i^T Q_i^{-1} \dddot{\alpha}_i \quad (29) \]

Then

\[ \dot{V} = r^T Dr + \frac{1}{2} r^T Dr + \sum_{i=1}^{n} \dddot{\theta}^T_i \Gamma_i^{-1} \ddot{\theta}_i + \sum_{i=1}^{n} \dddot{\alpha}_i^T Q_i^{-1} \dddot{\alpha}_i \quad (30) \]

In the light of the property (1) and property (2), the aboving equation can be revised as:

\[ \dot{V} = r^T (Dr + Cr) + \sum_{i=1}^{n} \dddot{\theta}^T_i \Gamma_i^{-1} \ddot{\theta}_i + \sum_{i=1}^{n} \dddot{\alpha}_i^T Q_i^{-1} \dddot{\alpha}_i \quad (31) \]

Putting the equation (28) into the aboving equation (31), the paper can get to the following equation:

\[ \dot{V} = -r^T K_r r - k_s r^T \text{sgn}(r) + \sum_{i=1}^{n} \dddot{\theta}^T_i \{\xi_i(q)\} \ddot{x}_i \]

\[ + \sum_{i=1}^{n} \dddot{\alpha}_i^T \{\varphi_i(z)\} \dddot{x}_i + r^T E + \sum_{i=1}^{n} \dddot{\theta}^T_i \Gamma_i^{-1} \ddot{\theta}_i \]

\[ + \sum_{i=1}^{n} \dddot{\alpha}_i^T Q_i^{-1} \dddot{\alpha}_i \quad (32) \]

Where

\[ r^T \{\dot{\Theta}^T \cdot \{\Xi(q)\}\} \ddot{y}_r = \sum_{i=1}^{n} \dddot{\theta}^T_i \{\xi_i(q)\} \ddot{x}_i \]

\[ r^T \{\dot{A}^T \cdot \{Z(z)\}\} \dot{y}_r = \sum_{i=1}^{n} \dddot{\alpha}_i^T \{\varphi_i(z)\} \dddot{x}_i \]

To put the adaptive law equation (24) and the equation (25) into the aboving equation, \( k_s > \|E\| \), so the paper can get to the following equation:

\[ \dot{V} \leq -r^T K_r r \leq 0 \quad (33) \]

From (33), and taking \( k_s > 0 \) into account. The paper can get to \( r \in L_2^\infty \). From the lemma, the paper can derive \( e \in L_2^\infty \cap L_\infty^\infty, \dot{e} \in L_2^\infty \), in which \( e \) is continuous. So when \( t \to \infty, e \to 0, r \to 0 \) and \( \dot{e} \to 0 \).

### 3. Simulation

About the free-floating space robot, the table 1 shows two-DOF space robot simulation parameters. The simulation environment Matlab is 7.0, simulation time is 20S.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_0 )</td>
<td>200 kg</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>0.85m</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>25 kg</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.6m</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>15 kg</td>
</tr>
<tr>
<td>( b_0 )</td>
<td>0.7m</td>
</tr>
<tr>
<td>( L_0 )</td>
<td>56 kg m²</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>0.85m</td>
</tr>
<tr>
<td>( I_1 )</td>
<td>2 kg m²</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.7m</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>1.8 kg m²</td>
</tr>
</tbody>
</table>

The desired trajectory of the end of space manipulator is:

\[ y_{d,1} = 1 + 0.3 \cos(\pi t) \]

\[ y_{d,2} = 1 + 0.3 \sin(\pi t) \]

The external interferences are:

\[ f = [q, 0.1 \sin t, q_1, 0.1 \sin t]^T \]

The filtered tracking error parameters are:

\[ \Lambda = \text{diag}(5, 5) \]

The controller gain:

\[ K_v = \text{diag}(10, 10); k_\gamma = 0.4 \]

The neural network initial weights are 0, the width of all basis functions are 10. The center of basis function is randomly selected in the input and output domain. Hidden nodes are 30 bits. The simulation results are shown in the Figure 2 ~ Figure 4.

The figure 2 shows the tracking scenario map of the position of manipulator’s end. The figure 3 shows the neural network approach situation map of the uncertain parts \( D(q) \) and \( C(q, \dot{q}) \) of the space robot. Where the approximate value is its norm number \( \| f(\cdot) \| \) . The figure 4 shows the situation of the control moment.
control torque is required to achieve better control precision, it is necessary to increase the control torque output. Considering that the space robots usually work under low speed condition in order to maintain their postures, the proposed control method can provide ample time for learning of neural network, and meet fully the requirement of real-time.

4. Conclusion

Aiming at the uncertainty of the free-floating space robot model in task space, this paper puts forward a neural network adaptive control method. This control method obtains control laws by the utilization of the neural network online modeling technology, the introduction of the GL matrix and its multiplication operator ". "., and the parameters through direct identification. The neural network approximation errors and external bounded disturbances are eliminated by sliding mode variable structure controller. The control method neither requires an estimate of inverse dynamic model, nor requires a time-consuming training process. Based on the Lyapunov theory, this control method proves global asymptotic stability of the whole closed-loop system. Simulation results show the neural controller can not only achieve higher precision without calculating the inverse Jacobian matrix, so it reduces the calculation quantity, but also meet real-time requirements. So it has great value in engineering applications.

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References


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