Decision Making Processes Supported by a Set Theoretical Approach

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Abstract

Current solutions for multi criteria decision making problems based on vague sets address static situations where a decision maker is presented with a number of alternatives and attributes. The attributes are placed in two disjoint sets related with 'OR' operation. The first set contains several attributes related with 'AND' operation while the second set contains a single attribute. We propose an extension of this approach that can handle situations where several decision makers are taking part in the decision making process and the second set of attributes consists of several elements.

Keywords: Set theory, multi criteria, multi person decision making.

1. Introduction

Multi-criteria fuzzy decision-making problems based on vague set theory have been originally addressed in [2]. The authors apply certain functions measuring the degree of suitability of each alternative. Since then a number of researchers have been working on these problems, see f. ex. [3], [6], [7].

Vague sets have been already exploitet for building models supporting multi criteria decision processes. These models are constructed to support the work of a single decision maker. This appears to be insufficient in complicated situations where expertise from different domains is required and a number of financially involved parties have to agree on the final decision. We propose an extension of this approach that can handle situations where several decision makers are taking part in the decision making and the second set of attributes consists of several elements.

When the universe of discourse $U$ is continuous, a vague set $A$ can be written as

$$A = \int [t_A(u_i), 1 - f_A(u_i)]/u_i$$

When the universe of discourse $U$ is discrete, a vague set can be written as

$$A = \sum_{i=1}^{n} [t_A(u_i), 1 - f_A(u_i)]/u_i$$

2. Background

Notations in this subsection are as in [6]. Let $U$ be the universe of discourse, $U = \{u_1, u_2, ..., u_n\}$ with a generic element of $U$ denoted by $u_i$. A vague set $A$ in $U$ is characterized by a truth-membership function $t_A$ and a false-membership function $f_A$,

$$t_A : U \rightarrow [0,1], \quad f_A : U \rightarrow [0,1],$$

where $t_A(u_i)$ is a lower bound on the grade of membership of $u_i$ derived from the evidence for $u_i$, $f_A(u_i)$ is a lower bound on the negation of $u_i$ derived from the evidence against $u_i$, and $t_A(u_i) + f_A(u_i) \leq 1$. The grade of membership of $u_i$ in the vague set $A$ is bounded to a subinterval $[t_A(u_i), 1 - f_A(u_i)]$ of $[0,1]$. The vague value $[t_A(u_i), 1 - f_A(u_i)]$ indicates that the exact grade of membership $\mu(u_i)$ of $u_i$ may be unknown. But it is bounded by $t_A(u_i) \leq \mu(u_i) \leq 1 - f_A(u_i)$, where $t_A(u_i) + f_A(u_i) \leq 1$.

When the universe of discourse $U$ is continuous, a vague set $A$ can be written as

$$A = \int [t_A(u_i), 1 - f_A(u_i)]/u_i$$

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**Definition 3** Let \( x \) and \( y \) be two vague values, \( x = [t_x, 1 - f_x] \) and \( y = [t_y, 1 - f_y] \), where \( t_x \in [0, 1], f_x \in [0, 1], t_y \in [0, 1], f_y \in [0, 1], t_x + f_x \leq 1 \) and \( t_y + f_y \leq 1 \). The result of the maximum operation of the vague values \( x \) and \( y \) is a vague value \( c \), written as \( c = x \vee y = [t_c, 1 - f_c] \), where

\[
t_c = \text{Max}(t_x, t_y), \quad 1 - f_c = \text{Max}(1 - f_x, 1 - f_y)
\]

Let \( A \) be a vague set of the universe of discourse \( U \) with truth-membership function and false-membership function \( t_A \) and \( f_A \), respectively, and let \( B \) be a vague set of \( U \) with truth-membership function and false-membership function \( t_B \) and \( f_B \), respectively. The notions of complement, union, and intersection of vague sets are defined as follows.

**Definition 6** The union of the vague sets \( A \) and \( B \) is a vague set \( C \), written as \( C = A \vee B \), whose truth-membership function and false-membership function are \( t_C \) and \( f_C \), respectively, where \( \forall u_i \in U \),

\[
t_C(u_i) = \text{Max}(t_A(u_i), t_B(u_i)), \quad 1 - f_C(u_i) = \text{Max}(1 - f_A(u_i), 1 - f_B(u_i))
\]

**Definition 7** The intersection of the vague sets \( A \) and \( B \) is a vague set \( C \), written as \( C = A \wedge B \), whose truth-membership function and false-membership function are \( t_C \) and \( f_C \), respectively, where \( \forall u_i \in U \),

\[
t_C(u_i) = \text{Min}(t_A(u_i), t_B(u_i)), \quad 1 - f_C(u_i) = \text{Min}(1 - f_A(u_i), 1 - f_B(u_i))
\]

## 3. Several Decision Makers

Decision making processes including several alternatives (objects) and a number of criteria (attributes) need to be considered also with respect to the number of decision makers being involved.

Suppose there are two disjoint sets of attributes \( A = \{A_i, i = 1, \ldots, n\} \) and \( B = \{B_i, i = 1, \ldots, m\} \), see Fig. 1.

![Figure 1: Two disjoint sets of attributes containing more than two elements each](image1)

This case can be handled by first applying AND operation in both sets \( A \) and \( B \) resulting in \( A' \) and \( B' \) and then apply OR operation on \( A' \) and \( B' \). Thus, if

\[
(A_1 \text{ AND } A_2 \text{ AND } \ldots \text{ AND } A_n) = A',
\]

\[
(B_1 \text{ AND } B_2 \text{ AND } \ldots \text{ AND } B_m) = B'
\]

then the outcome is \( (A' \text{ OR } B') \). A situation requires involvement of more than two disjoint sets of attributes will be hold in an analogous fashion, i.e.

\[
(A' \text{ OR } B' \text{ OR } \ldots \text{ OR } S')
\]

Suppose there are two intersecting sets of attributes \( A = \{A_1, \ldots, A_s, E_1, \ldots, E_k\} \) and \( B = \{B_1, \ldots, B_t, E_1, \ldots, E_k\} \), see Fig. 2.

![Figure 2: Two intersecting sets of attributes](image2)

In this case we propose to proceed in a way similar to the one described above. Thus we are looking at three disjoint sets,

\[
(A_1 \text{ AND } A_2 \text{ AND } \ldots \text{ AND } A_s) = A',
\]

\[
(B_1 \text{ AND } B_2 \text{ AND } \ldots \text{ AND } B_t) = B'
\]

and

\[
(E_1 \text{ AND } E_2 \text{ AND } \ldots \text{ AND } E_k) = E'
\]
The outcome is \( A' \text{ OR } B' \text{ OR } E' \). If a situation requires involvement of more than two intersecting sets of attributes will be hold in an analogous fashion.

In the case of \( q > 1 \) where the decision makers are with different degree of influence it is advisable that they work out a set of coefficients addressing these differences, i.e. \( 0 < w_i < 1, i = 1, ..., q \), where

\[
\sum_{i=1}^{q} w_i = 1.
\]

The outcome is then \( A' \times w_1 \text{ OR } B' \times w_2 \) for two disjoint sets of attributes. The case with intersecting sets will be \( A' \times w_1 \text{ OR } B' \times w_2 \text{ OR } E' \) since the attributes in \( E' \) are of interest to all decision makers.

4. Conclusions

Multi criteria decision making supported by vague sets has been presented. An approach applicable for involvement of several decision makers has been presented. The approach extends an existing one where a single decision maker is assumed.

References


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