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Fuzzy quadratic multiobjective portfolio selection model: a possibility approach

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Abstract

This paper develops a quadratic multiobjective model with fuzzy returns of the portfolio selection problem (QMPS). The obtained fuzzy model cannot be efficiently solved using traditionally approaches. A possibility approach is introduced in which objectives are treated as fuzzy events. The approach transforms the fuzzy QMPS model into a possibility QMPS problem by using possibility measures of fuzzy events. A particle swarm optimization algorithm is used to solve the crisp quadratic problem obtained. A numerical example is provided to demonstrate the effectiveness of the solution approach and the model efficiency.

Keywords: *fuzzy portfolio selection, quadratic multiobjective problem, Possibility theory, Particle swarm optimization.*

1. Introduction

Portfolio selection has a central role in finance theory and practical applications. It has been first developed on the basis of a mean-variance approach proposed by [1], who combines probability theory and optimization theory to model the behavior of the investor. The aim of the mean-variance model is to use the expected return of a portfolio as the investment return and the expected variance of the portfolio as the investment risk. In recent portfolio problem, other criteria have been considered to perform the decision maker choices and for this reason, multiple-criteria models have been proposed in [2], [3] and [4].

Traditionally, portfolio selection models are based on crisp variables and parameters. In other words, these models represent the situation of an investor who has all of the information that is necessary for decision making. However, the information available in financial markets is often incomplete, and thus, decisions are made under uncertainty. Additionally, markets are affected by vagueness and ambiguity caused by the use of linguistic terms such as ‘high risk’, ‘low profit’ and ‘low liquidity’ by the investors and the investment experts. Consequently, fuzzy set theory [5] represents an interesting alternative to deal with subjective preferences of investors and expert knowledge in portfolio selection problem. Numerous portfolio selection models with fuzzy parameters are proposed. For example, [6], [7] and [8] proposed different possibilistic mean-variance models. Based on credibility measure, [9] developed fuzzy mean-variance models and further fuzzy mean-semi variance models [10]. [11] proposed fuzzy mean-variance-cross entropy models. [12] proposed fuzzy mean-variance-skewness models. A review of the fuzzy portfolio selection can be found in [13].

In this paper, we develop a quadratic multiobjective portfolio selection model which aims to maximize fuzzy long-term and short-term return and minimize the covariance. We use a possibility approach to treat fuzzy objectives in the proposed model. Following this approach, fuzzy quadratic multiobjective portfolio selection model (FQMPS) is transformed into possibility quadratic programming problem. For the case of fuzzy parameters with trapezoidal membership functions, possibility QMPS model becomes a crisp quadratic multiobjective problem, and can be solved by some evolutionary algorithms.

The paper is organized as follows. The quadratic multiobjective portfolio selection model is introduced in Section 2. Section 3 presents the possibility approach to treat fuzzy parameters in the FQMPS model. In Section 4, the particle swarm optimization technique is described. Section 5 discusses a numerical experiment in determining the optimal portfolio when trapezoidal fuzzy parameters are used. Finally, Section 6 concludes the paper, and discusses some future research directions.

2. Mathematical modeling

Several different mathematical optimization approaches have been described for the portfolio

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optimization problem in [14]. In this study, In order to construct the mathematical model for the portfolio selection, we firstly introduce the following notations

α_i the fuzzy short-term return of the ith asset,

β_i the fuzzy long-term return of the ith asset,

r_1 the lower limit on the expected short-term return of the portfolio,

r_2 the lower limit on the expected long-term return of the portfolio,

σ_{ij} the covariance between assets i *and* j ,

N the available number of assets,

h the number of assets to invest ($h \le N$),

n_i the minimum inversion ratio allowed in the *i*th asset,

m_i the maximum inversion ratio allowed in the *i*th asset,

x_i the proportion of the total funds invested in the ith asset,

We define also the variable

$$z_i = \begin{cases} 1 & \text{if the ith (i=1,...,N) asset is chosen} \\ 0 & \text{otherwise} \end{cases}$$

The quadratic multiobjective portfolio selection model with fuzzy returns (P1) (FQMPS) is formulated as

$$\begin{aligned} &\max\ f_1(x) = \alpha_1 x_1 + \alpha_2 x_2 + ...+ \alpha_n x_n \ge r_1 \\ &\max\ f_2(x) = \beta_1 x_1 + \beta_2 x_2 + ...+ \beta_n x_n \ge r_2 \qquad (1) \\ &\min\ f_3(x) = \sum_{i=1}^{n} x_i x_j \sigma_{ij} \end{aligned}$$

Subject to

$$\sum_{i=1}^{n} x_i = 1 \qquad (2)$$

$$x_i \le m_i z_i \qquad (3)$$

$$x_i \ge n_i z_i \qquad (4)$$

$$\sum_{i=1}^{n} z_i = h \qquad (5)$$

$$z_i \in \{0,1\} \qquad (6)$$

$$x_i \ge 0,\ i = 1,......,n$$

Model (P1) takes the form of quadratic fuzzy multiobjective problem. It contains three objective functions. The first consists to maximize the short-term return with respect to the condition of lower limit on the expected return of the portfolio. The second aims to maximize the long-term return with respect to the condition of lower limit on the expected return of portfolio. The third aims to minimize the covariance between returns of assets i and j .

Eq. (2) ensures that the sum of the proportions is 1. Constraints (3) and (4) ensure that if any of asset i is held, its proportion x_i must lie between n_i and m_i . Eq. (5) ensures that exactly h assets are held. Eq. (6) is the integrality constraint.

In the next sections we present a new possibility approach to solving FPS models. This approach provides a way to deal with the uncertainty in fuzzy objectives through the use of possibility measures.

3. Possibility approach

Possibility theory was formulated in terms of fuzzy set theory by [15] and has been developed by many researchers. A good reference on possibility theory can be found in [16]. Zadeh suggested that fuzzy sets can be used as a basis for the theory of possibility similar to the way that measure theory provides the basis for the theory of probability. He introduced the “fuzzy variable”, which is associated with a possibility distribution in the same manner that a random variable is associated with a probability distribution. In the FQMPS model, each fuzzy coefficient can be viewed as a fuzzy variable and each constraint or objective can be considered as fuzzy event. Using possibility theory, possibilities of fuzzy events (i.e., fuzzy constraints or objectives) can be determined.

3.1 Fuzzy event via possibility measure

Let $(\Theta_i, \mathrm{P}(\Theta_i), \pi_i)$, for each $i = 1,2,...,n$, be a possibility space with Θ_i being the nonempty set of interest, $\mathrm{P}(\Theta_i)$ the collection of all subsets of Θ_i , and π_i the possibility measure from $\mathrm{P}(\Theta_i)$ to $[0,1]$.

Given a possibility space $(\Theta_i, \mathrm{P}(\Theta_i), \pi_i)$ with

- $\pi(\varnothing) = 0$, $\pi(\Theta_i) = 1$ and
- $\pi\left(\bigcup A_i\right) = \sup_i \{\pi(A_i)\}$ with each $A_i \in \mathrm{P}(\Theta_i)$,

Zadeh defined a fuzzy variable, $\digamma$, as a real-valued function defined over Θ_i with the membership function

$$\begin{aligned} \mu_{\digamma}(s) &= \pi\left(\{\theta_i \in \Theta_i / \digamma(\theta_i) = s\}\right) \\ &= \sup_{\theta_i \in \Theta_i} \{\pi(\{\theta_i\}) / \digamma(\theta_i) = s\},\ s \in R \end{aligned}$$

Let $(\Theta, \mathrm{P}(\Theta), \pi)$ be a product possibility space such that $\Theta = \Theta_1 \times \Theta_2 \times .... \times \Theta_n$ and from possibility theory ,

$$\pi(A) = \min_{i=1,2,\dots,n} \{\pi(A_i) / A = A_1 \times A_2 \times \dots \times A_n,\ A_i \in \mathrm{P}(\Theta_i)\}$$

Suppose $\tilde{a}$ and $\tilde{b}$ are two fuzzy variables on the possibility spaces $(\Theta_1, \mathrm{P}(\Theta_1), \pi_1)$ and $(\Theta_2, \mathrm{P}(\Theta_2), \pi_2)$, respectively. The relation $\tilde{a} \le \tilde{b}$ is a fuzzy event defined on the product possibility space $(\Theta = \Theta_1 \times \Theta_2, \mathrm{P}(\Theta), \pi)$, with

$$\pi(\tilde{a} \le \tilde{b}) = \sup_{\theta_1,\theta_2} \{\pi\{(\theta_1,\theta_2) / \tilde{a}(\theta_1) \le \tilde{b}(\theta_2)\}, \theta_1 \in \Theta_1, \theta_2 \in \Theta_2\}$$
$$= \sup_{\theta_1,\theta_2} \{\min\{\pi(\theta_1), \pi(\theta_2)\} / \tilde{a}(\theta_1) \le \tilde{b}(\theta_2), \theta_1 \in \Theta_1, \theta_2 \in \Theta_2\}$$

Furthermore, from the definition of fuzzy variables, we have

$$\pi(\tilde{a} \le \tilde{b}) = \sup_{s,t \in R} \{\min\{\mu_{\tilde{a}}(s), \mu_{\tilde{b}}(t)\} / s \le t\}$$

Similarly, possibilities of the fuzzy events $\tilde{a} < \tilde{b}$ and $\tilde{a} = \tilde{b}$ defined on the product possibility space $(\Theta, \mathrm{P}(\Theta), \pi)$ are given as

$$\pi(\tilde{a} < \tilde{b}) = \sup_{s,t \in R} \{\min\{\mu_{\tilde{a}}(s), \mu_{\tilde{b}}(t)\} / s < t\}$$

$$\pi(\tilde{a} = \tilde{b}) = \sup_{s,t \in R} \{\min\{\mu_{\tilde{a}}(s), \mu_{\tilde{b}}(t)\} / s = t\}$$

when the right hand side $\tilde{b}$ becomes a crisp value $\tilde{b}$, then the possibilities of the corresponding fuzzy events are given as

$$\pi(\tilde{a} \le b) = \sup_{s \in R} \{\mu_{\tilde{a}}(s) / s \le b\}$$

$$\pi(\tilde{a} < b) = \sup_{s \in R} \{\mu_{\tilde{a}}(s) / s < b\}$$

$$\pi(\tilde{a} = b) = \mu_{\tilde{a}}(b)$$

Let $\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$ be fuzzy variables and $f_j : R^n \to R$ be a real-valued function, for $j = 1, \dots, m$. The possibility of the fuzzy event « $f_j(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \le 0, \quad j = 1, \dots, m$ » is given by

$$\pi(f_j(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \le 0, \quad j = 1, \dots, m)$$
$$= \sup_{s_1,\dots,s_n \in R} \left\{ \min_{1 \le i \le n} \{\mu_{\tilde{a}_i}(s_i)\} / f_j(s_1, s_2, \dots, s_n) \le 0, \quad j = 1, \dots, m \right\}$$

3. 2 Possibilistic quadratic multiobjective problem

The concept of chance-constrained programming (CCP), which was introduced by [17], is adopted in this paper as a way to solve the FQMPS model. CCP deals with uncertainty by specifying the desired levels of confidence with which the objectives hold. Using the concepts of CCP and possibility of fuzzy events, the FQMPS model becomes the following possibility problem (P2)

$$\max\ f_1 = \pi(\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n \ge r_1) \ge a_1$$
$$\max\ f_2 = \pi(\beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n \ge r_2) \ge a_2$$
$$\min\ f_3 = \sum_{i=1}^{n} x_i x_j \sigma_{ij}$$

Subject to

$$\sum_{i=1}^{n} w_i = 1$$
$$w_i \le m_i z_i$$
$$w_i \ge n_i z_i$$
$$z_i \in \{0,1\},\ \sum_{i=1}^{n} z_i = h,\ w_i \ge 0,\ i = 1, \dots, n$$

where a_1 and $a_2 \in [0,1]$ are the acceptable levels of possibility for the first and the second objective, respectively.

Definition 1 (Normal fuzzy variables). Given a fuzzy variable $\tilde{a}$ on a possibility space $(\Theta, \mathrm{P}(\Theta), \pi)$ the fuzzy variable $\tilde{a}$ is normal if

$$\sup_{s \in R} \mu_{\tilde{a}}(s) = 1$$

Definition 2 (α-level set). The α-level set of a fuzzy variable $\tilde{a}$ is defined by the set of elements that belong to the fuzzy variable $\tilde{a}$ with membership of at least α, i.e.,

$$a_\alpha = \{s \in R / \mu_{\tilde{a}}(s) \ge \alpha\}$$

Definition 3 (Convex fuzzy variables). A fuzzy variable is convex $\tilde{a}$ if

$$\mu_{\tilde{a}}(\lambda s_1 + (1-\lambda) s_2) \ge \min(\mu_{\tilde{a}}(s_1), \mu_{\tilde{a}}(s_2))$$
$$,\quad s_1, s_2 \in R, \quad \lambda \in [0,1]$$

Alternatively, the fuzzy variable $\tilde{a}$ is convex if all α-level sets are convex.

Lemma 1. Let $\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$ be fuzzy variables with normal and convex membership functions.

Let $(.)_{\alpha_i}^L$ and $(.)_{\alpha_i}^U$ denote the lower and upper bounds of the α-level set of $\tilde{a}_i, i = 1, \dots, n$. Then, for any given possibility levels $\alpha_1, \alpha_2, \alpha_3$ with $0 \le \alpha_1, \alpha_2, \alpha_3 \le 1$,

$\pi(\tilde{a}_1 + \tilde{a}_2 + \dots + \tilde{a}_n \le b) \ge \alpha_1$ if and only if $(\tilde{a}_1)_{\alpha_1}^L, \dots, (\tilde{a}_n)_{\alpha_1}^L \le b$,

$\pi(\tilde{a}_1 + \tilde{a}_2 + \dots + \tilde{a}_n \ge b) \ge \alpha_2$ if and only if $(\tilde{a}_1)_{\alpha_2}^U, \dots, (\tilde{a}_n)_{\alpha_2}^U \ge b$,

$\pi(\tilde{a}_1 + \tilde{a}_2 + \dots + \tilde{a}_n = b) \ge \alpha_3$ if and only if $(\tilde{a}_1)_{\alpha 3}^L, \dots, (\tilde{a}_n)_{\alpha_3}^L \le b$ and $(\tilde{a}_1)_{\alpha_3}^U, \dots, (\tilde{a}_n)_{\alpha_3}^U \ge b$

From Lemma 1 as well as Liu [17], for a trapezoidal fuzzy number $((\tilde{r}_i)_0^L, (\tilde{r}_i)_1^L, (\tilde{r}_i)_0^U, (\tilde{r}_i)_1^U)$ and any given possibility level $\alpha, 0 \le \alpha \le 1$, the following are true:

$$\pi\left(\tilde{r}_1 + \ldots + \tilde{r}_n \le b\right) \ge \alpha \text{ if and only if}$$
$$(1-\alpha)\left(\left(\tilde{r}_1\right)_0^L + \ldots + \left(\tilde{r}_n\right)_0^L\right) + \alpha\left(\left(\tilde{r}_1\right)_1^L + \ldots + \left(\tilde{r}_1\right)_1^L\right) \le b$$
$$\pi\left(\tilde{r}_1 + \ldots + \tilde{r}_n \ge b\right) \ge \alpha \text{ if and only if}$$
$$(1-\alpha)\left(\left(\tilde{r}_1\right)_0^U + \ldots + \left(\tilde{r}_n\right)_0^U\right) + \alpha\left(\left(\tilde{r}_1\right)_1^L + \ldots + \left(\tilde{r}_n\right)_1^U\right) \ge b$$

Therefore, when inputs and outputs are trapezoidal fuzzy numbers, the possibilistic QMPS problem becomes the following crisp quadratic multiobjective model (P3)

$$\max f_1(x) = (1-a_1)\left(\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n\right)_0^U$$
$$+ a_1\left(\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n\right)_1^U$$
$$\max f_2(x) = (1-a_2)\left(\beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n\right)_0^U$$
$$+ a_2\left(\beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n\right)_1^U$$
$$\min f_3(x) = \sum_{i=1}^{n} x_i x_j \sigma_{ij}$$

Subject to

$$(1-a_1)\left(\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n\right)_0^U + a_1\left(\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n\right)_1^U \ge r_1$$
$$(1-a_2)\left(\beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n\right)_0^U + a_2\left(\beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n\right)_1^U \ge r_2$$
$$\sum_{i=1}^{n} x_i = 1$$
$$x_i \le m_i z_i$$
$$x_i \ge n_i z_i$$
$$z_i \in \{0,1\}, \sum_{i=1}^{n} z_i = h, x_i \ge 0, \; i = 1, \ldots, n$$

4. Particle swarm optimization technique

Particle Swarm Optimization (PSO) is a population optimization algorithm inspired by social behavior of bird flocking. It belongs to Swarm Intelligence, which originates from the study of natural creatures living in a group. Each individual possess little or no wisdom, but by interacting with each other or the surrounding environment, they can perform very complex tasks as a group.

The PSO algorithm starts with the initialization of a population of random particles, each of which is associated with a position and a velocity. The velocities are adjusted according to the historical behavior of each particle and its neighbors while they fly through the search space. The positions are updated according the current position and the velocities at the next step. Therefore, the particles have a tendency to fly towards the better and better search area over the search process course. Each particle tries to modify its position using the current positions, the current velocities, the distance between the pbest and the current position and the distance between the gbest and the current position.

4. 1 Fitness function

[18] suggested a fitness value associated with each particle. Thus, a particle moves in solution space with respect to its previous position where it has met the best fitness value, and the neighbor's previous position where the neighbor has met the best fitness value. In this study, the fitness function is defined as

$$f_p = d_1 f_1(x) + d_2 f_2(x) - d_3 f_3(x)$$

where f_p is the fitness value of particle p and $d_j \,(j = 1,2,3)$ is a weight reflecting the relative importance of the jth objective. The sum of these weights is equal to one.

4. 2 Moving a particle

At each iteration t, the position x_{ij}^t of the ith particle is updated by a velocity v_{ij}^{t+1}. The position is updated for the next iteration using the following formula

$$x_{ij}^{t+1} = x_{ij}^t + v_{ij}^{t+1}$$

where x_{ij}^t denotes the position of particle i in the dimension j search space at time step t. The position of the particle is changed by adding a velocity v_{ij}^{t+1} to the current position. The velocity update rule is calculated as

$$v_{ij}^{t+1} = v_{ij}^t + c_1 r_1\left(p_{ij} - x_{ij}^t\right) + c_2 r_2\left(p_{gj} - x_{ij}^t\right)$$

In this formula, v_{ij}^t is the velocity of particle i in dimension j = 1, . . . , n at time step t. The personal best position, p_{ij} associated with particle i in dimension j, is the best position the particle has visited since the first time step. The global best position p_{gj} at time step t is the best position discovered by all particles found since the first time step. The values r_1 and r_2 are random in the range [0, 1] and sampled from an uniform distribution. These random values introduce a stochastic element to the algorithm. The positive acceleration coefficients c_1 and c_2 are used to scale the contribution of the cognitive and social components, respectively.

To improve PSO convergence, [19] proposed a strategy for incorporating inertial weight w as a mechanism for controlling swarm exploitation and exploration by weighting the contribution of the previous velocity. This weight control how much

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the memory of the previous flight direction influences the new velocity. For $w \geq 1$, velocity increase over time, accelerates to maximum velocity, and the swarm diverges. Particle fails to change direction to move back towards promising areas. For $w < 1$, particles decelerate until their velocity is zero. The velocity update with inertia is given as

$$v_{ij}^{t+1} = wv_{ij}^{t} + c_1 r_1 \left(p_{ij} - x_{ij}^{t}\right) + c_2 r_2 \left(p_{gj} - x_{ij}^{t}\right)$$

In this study we use the following parameters

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{itr_{\max}} \times itr$$

$$c_1 = c_{1\max} - \frac{c_{1\max} - c_{1\min}}{itr_{\max}} \times itr$$

$$c_2 = c_{2\max} - \frac{c_{2\max} - c_{2\min}}{itr_{\max}} \times itr$$

4. 3 Constraint satisfaction

For handling the cardinality constraints, h is the desired number of assets in the portfolio. Given a set Q of h assets, let h' represent the number of assets after updating positions in portfolio (the numbers of the proportion x_i greater than 0). If $h' < h$, then some assets must be added to Q; if $h' > h$, then some assets must be removed from Q until $h' = h$.

Considering the removal of assets in the case where $h' > h$, we delete the smallest assets. If $h' < h$, assets remaining to be added must be identified. In this study, we randomly add an asset $i \notin Q$ and assign the minimum proportional value ε_i to the new asset.

The value of x_i must also satisfy $0 \leq n_i \leq x_i \leq m_i$ for $i \in Q$. Let s_i represent the proportion of the new position belonging to Q. If $s_i < n_i$, the minimum proportional value of n_i replaces asset s_i. If $s_i > n_i$, the proportional share of the free portfolio is calculated as follows

$$x_i = n_i + \frac{s_i}{\sum_{j \in Q, s_i > n_i} s_i} \left(1 - \sum_{j \in Q}^{n_i} 1\right)$$

This minimizes the proportional value of n_i for the useless assets $i \in Q$ so that particles converge faster in the search process.

The following is a summary of the PSO algorithm steps.

1) Initialize particles with random position and velocity vectors.
2) For each particle's position "P", evaluate the fitness.
3) Compare particle's fitness (P) with fitness (pbest). If P is greater than pbest then p= pbest.
4) Set best of pbest as gbest.
5) Update particle's velocity and position.
6) Stop giving gbest as the optimal solution.

5. Numerical experiments

We use in this study data concerning short-term return and long term for 8 assets listed in stock exchange of Tunis, Tunisia. These data are represented by fuzzy trapezoidal numbers (a;b;c;d), where b et c are the center values and a and d are the left endpoint and right endpoint, respectively.

Table 1: imput data

Assets	***Short-term return***	***Long-term return***
A1	(-0.26. ; -0.13 : 0.63 ; 0.637)	(-0.38 ; -0.19 : 0.26 ; 0.44)
A2	(-0.63. ; -0.363 : 0.53 ; 0.63)	(-0.86 ; -0.69 : 0.27 ; 0.54)
A3	(-0.83 ; -0.67 : 0.28 ; 0.49)	(-0.33. ; -0.26 : 0.63 ; 0.69)
A4	(-0.88 ; -0.69 : 0.27 ; 0.44)	(-0.51 ; -0.33 : 0.27 ; 0.48)
A5	(-0.81 ; -0.72: 0.29 ; 0.48)	(-0.72 ; -0.49 : 0.22 ; 0.39)
A6	(-0.89 ; -0.61: 0.31 ; 0.44)	(-0.88 ; -0.69 : 0.27 ; 0.44)
A7	(-0.66 ; -0.59 : 0.29 ; 0.48)	(-0.89 ; -0.72 : 0.37 ; 0.43)
A8	(-0.87 ; -0.66 : 0.37 ; 0.45)	(-0.81 ; -0.29 : 0.47 ; 0.54)

We solve five portfolio selection problems for different possibility levels (0, 0.25, 0.50, 0.75, 1) with PSO and GA algorithms. As termination condition, we use 100 steps in the PSO solution and 100 generations in the GA solution. The primary attributes of the problems solved by these algorithms are summarized in Table 2.

Table 2: primary attributes of the problems

	PSO	**GA**
r_1	0.12	0.12
r_2	0.15	0.15
h	5	5
TC	100 steps	100 generations

The values of the objective functions of the portfolio selection model that correspond to the best fitness found by the PSO and GA are presented in Table 3. The results obtained by the PSO algorithm are different from those obtained by the GA at all possibility levels. We remark that the values of short and long terms are higher with the PSO. However, for the third objective, the values given by the GA are lower and so that they are the better since that the goal is to minimize the risk represented by the covariance. This difference is due to the investor's attitude. If the decision maker is risk-loving, the sum of weights affected in the fitness function to the return objectives is greater than the weight assigned to the risk objective. Then we can say that PSO outperform the GA if the investor is risk-loving and the GA is the better in the case of a risk-averse investor.

In addition, as shown in tables 3 and 4, the value of the objective return is an increasing function of the possibility level and that can be explain by the fact that the goal is to maximize the returns subject to a set of linear constraints.

Table 3: values of the various objective functions for PSO at different possibility levels

h	**Short term return**	**long term return**	**covariance**
0	0.6724	0.7987	0.3356
0.25	0.6726	0.7794	0.3328
0.5	0.6731	0.8724	0.2723
0.75	0.6750	0.8524	0.2717
1	0.6752	0.8724	0.1724

Table 4: values of the various objective functions for GA at different possibility levels

h	**Short term return**	**long term return**	**covariance**
0	0.6324	0.7283	0.2356
0.25	0.6425	0.7794	0.2328
0.5	0.6432	0.8725	0.1724
0.75	0.6650	0.8734	0.1715
1	0.6752	0.8744	0.1704

Tables 5 and 6 demonstrate that the optimal portfolio obtained with the PSO algorithm at the possibility level 0.5 is the combination of the five assets A1, A2, A4, A5 and A7. At the same possibility level, these assets are A1, A4, A5, A7 and A8 when the GA is used. At the possibility level 0.75, the structure of the optimal portfolio changes for the two algorithms. Here, there's not a clear relation between the possibility level and the structure of the optimal portfolio.

In addition, we cannot decide about the effectiveness of the two algorithms and in this case, the key criterion of decision is the investor's satisfaction level. It is also interesting to note that the solution at each possibility level serves as a scenario for the decision maker.

Table 5: The asset allocation at the possibility level 0.5

Assets	***PSO***	***GA***
A1	0.125	0.211
A2	0.118	0
A3	0	0
A4	0.342	0.2990
A5	0.08	0.2000
A6	0	0
A7	0.3259	0.24009
A8	0	0.051

Table 6: The asset allocation at the possibility level 0.75

Assets	***PSO***	***GA***
A1	0.125151	0.211
A2	0	0.299
A3	0.31880	0
A4	0.34	0.1990
A5	0.0900	0.1500
A6	0	0
A7	0.1249	0.1400
A8	0	0

6. Conclusion

In this paper, a possibility approach for solving fuzzy quadratic multiobjective portfolio selection model has been developed. In this approach, fuzzy objectives are defined by possibility measures. For the case of fuzzy returns with trapezoidal membership functions, the possibility QMPS model turns out to be a crisp quadratic multiobjective problem. A numerical study concerning 8 assets listed in stock exchange of Tunis was used to demonstrate the implementation and interpretation of the results from the possibility approach.

Results obtained by the PSO algorithm are given to compare with those obtained by the GA. As mentioned in the last section, at a possibility level, α , the structure of the optimal portfolio is not the same for the two algorithms and the efficiency of an algorithm relative to each other depends to the investor preferences. Also, the value of an objective is an increasing function of the possibility level when the goal is to maximize the return and a decreasing in the case of minimizing the risk. In addition, the fraction of the capital budget invested in each asset depends on the possibility level and this provides the flexibility to decision makers to set their own acceptable (possibility) levels in selecting their appropriate optimal portfolio.
The possibility approach should enhance the capability of decision makers to improve their operations in a competitive, vague and uncertain environment. Another interesting topic for future work is the solution of possibility portfolio selection models with general membership functions.

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