

Directional Total Variation Filtering Based Image Denoising Method

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Abstract

In this paper, we study signal denoising technique based on total variation (TV) which was reported by Ivan W. Selesnick, Ilker Bayram [1]. Here, we present a directional total variation algorithm for image denoising. In most of the image denoising methods, the total variation denoising is directly performed on the noisy images. In this work, we apply a 1D TV denoising algorithm in sequential manner on the pixel sequence obtained in different orientations including zig-zag, horizontal and vertical. The performance of the proposed method is evaluated using the standard test images and the quality of the denoised images is assessed using various objective metrics such as peak signal to noise ratio (PSNR), structural similarity index metrics (SSIM), visual signal to noise ratio (VSNR). Various experiments show that the proposed method provides promising results with less computational load.

Keywords: Total Variation, maximization-minimization, min-max, zig-zag.

1. Introduction

Typically images are piecewise constant which are retained in Total Variation Denoising. The denosing problem can be thought of as TV reduction problem. Archives and libraries contain many documents that have images with defects due to spots, aging etc. The distortions occurring to the images can restrict its quality during image segmentation, transmission, storage etc. There are different types of noises such as gaussian, localvar, poisson, salt and pepper, speckle, which can be added to an image through MATLAB for experiments. Average filtering can reduce the additive Gaussian noise. Our paper helps by explaining image denosing using Total Variation. Rudin, Osher, and Fatemi [3] introduced Total Variation based filtering. The maximization-minimization procedure along with min-max property is being used. By [4],[5] (Chambolle's Algorithm), considering an image of size 256x256 we require two matrixes with same size as

image size. This leads to a constraint on memory sensitive devices like those that on the mobile environment. The proposed method on the other hand requires only one matrix of size 256x256. This reduces the size by half the actual length required.

In literature, many denoising algorithms have been proposed based on linear and non linear filtering methods. For example, median filtering method removes salt and pepper noise present in the image meanwhile it smooths out edges in the image. Recently, total variation method shows that it simultaneously removes the noise pixels and preserves the edges. In all the methods, the total variational filtering is directly applied on the images. In this work, we apply the TV filtering method on the pixel sequence that is obtained for different orientations like vertical, horizontal and zig-zag. The rest of this paper is as follows. In Section 2, we provide a short summary of total variational filtering method reported in [1]. In Section 3, we present our proposed method based on the TV and directional filtering concepts. In Section 4, we discuss different image quality assessment techniques. In Section 5, we discuss experimental results obtained for standard test images with different signal to noise ratio. We also present results of quality assessment performed using various objective metrics, namely, signal to noise ratio (SNR), mean square error (MSE), peak signal to noise ratio (PSNR), laplacian mean square error (LMSE), normalized absolute error (NAE), structural similarity index metrics SSIM), visual signal to noise ratio (VSNR) and singular value decomposition based image quality assessment (SVD-IQA). Finally, we provide concluding remarks in Section 6.

2. Total Variational Filtering

The total-variation (TV) of a signal measures how much the signal changes between signal values. Specifically, the

total-variation of an N -point signal $x(n), 1 \leq n \leq N$ is defined as

$$TV(x) = \sum_{n=2}^N |x(n) - x(n-1)| \quad (1)$$

The total variation of x can also be written as

$$TV(x) = Dx_{\Gamma_1} \quad (2)$$

Where

$$D = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & & \ddots & \\ & & & & -1 & 1 \end{bmatrix} \quad (3)$$

is a matrix of size $(N-1) \times N$. We assume we observe the signal x corrupted by additive white Gaussian noise,

$$y = x + n, \quad y, x, n \in R^N \quad (4)$$

One approach to estimate x is to find the signal x minimizing the objective function

$$J(x) = y - x_{\Gamma_2}^2 + \lambda_{\Gamma} Dx_{\Gamma_1} \quad (5)$$

This approach is called TV denoising. The regularization parameter λ , controls how much smoothing is performed. Larger noise levels call for larger λ .

2.1 Algorithm for TV Denoising

We will assume a more general form of the objective function:

$$J(x) = y - x_{\Gamma_2}^2 + \lambda_{\Gamma} Ax_{\Gamma_1} \quad (6)$$

where A is a matrix of size $M \times N$. The optimal value of the objective function is denoted

$$J_* = \min_x \Gamma y - x_{\Gamma_2}^2 + \lambda_{\Gamma} Ax_{\Gamma_1} \quad (7)$$

The minimization of this objective function is complicated by the fact that the ℓ_1 norm is not differentiable. Therefore, an approach to minimize $J(x)$ is to use the dual formulation. To derive the dual formulation, note that the absolute value of a scalar x can be written in the following circuitous form:

$$|x| = \max_{|z| \leq 1} zx \quad (8)$$

The advantage of this form is that the nonlinearity of the function is transferred to the feasible set. Likewise, note that the ℓ_1 norm of a vector x can be written as:

$$\Gamma x_{\Gamma_1} = \max_{|z| \leq 1} z^T x \quad (9)$$

Where the condition $|z| \leq 1$ is taken element-wise. Similarly,

$$\Gamma Ax_{\Gamma_1} = \max_{|z| \leq 1} z^T Ax \quad (10)$$

Therefore, we can write the objective function $J(x)$ in (6) as

$$J(x) = y - x_{\Gamma_2}^2 + \lambda \max_{|z| \leq 1} z^T Ax \quad (11)$$

Or

$$J(x) = \max_{|z| \leq 1} \Gamma y - x_{\Gamma_2}^2 + \lambda z^T Ax \quad (12)$$

The optimal value of the objective function (7) is

$$J_* = \min_x \max_{|z| \leq 1} \Gamma y - x_{\Gamma_2}^2 + \lambda z^T Ax \quad (13)$$

We want to find the vector x giving this value, but it will be convenient to find both x and the auxiliary vector z .

Defining

$$F(x, z) := y - x_{\Gamma_2}^2 + \lambda z^T Ax \quad (14)$$

We can write:

$$J_* = \min_x \max_{|z| \leq 1} F(x, z) \quad (15)$$

Because $F(x, z)$ is convex in x and concave in z , the optimal value J_* is a saddle point of $F(x, z)$. By the *min-max* property, we can write:

$$J_* = \max_{|z| \leq 1} \min_x F(x, z) \quad (16)$$

Or

$$J_* = \max_{|z| \leq 1} \min_x \Gamma y - x_{\Gamma_2}^2 + \lambda z^T Ax \quad (17)$$

which is the dual formulation of the TV denoising problem. The inner minimization problem in (17) can be solved as follows:

$$\frac{\partial}{\partial x} F(x, z) = -2(y - x) + \lambda A^T z \quad (18)$$

$$\text{So } \frac{\partial}{\partial x} F(x, z) = 0 \Rightarrow x = y - \frac{\lambda}{2} A^T z \quad (19)$$

Substituting (19) back into (17) gives

$$J_* = \max_{|z| \leq 1} \Gamma \left(\frac{\lambda}{2} A^T z_{\Gamma_2}^2 + \lambda z^T A \left(y - \frac{\lambda}{2} A^T z \right) \right) \quad (20)$$

After simplifying we have,

$$J_* = \max_{|z| \leq 1} -\frac{\lambda^2}{4} z^T A A^T z + \lambda z^T A y \quad (21)$$

Or equivalently, the minimization problem:

$$z_* = \arg \min_{|z| \leq 1} z^T A A^T z - \frac{4}{\lambda} z^T A y \quad (22)$$

Setting the derivative with respect to z as zero gives the equation

$$A A^T z = \frac{2}{\lambda} A y \quad (23)$$

which requires the solution to a potentially large system of linear equations and furthermore does not yield a solution z satisfying the constraint $|z| \leq 1$. To find z solving the constrained minimization problem(22), the majorization-minimization (MM) method can be used[2]

Defining

$$D(z) = z^t A A^t z - \frac{4}{\lambda} z^t A y \quad (24)$$

and setting $z^{(i)}$ as point of coincidence, we can find a separable majorizer of $D(z)$ by adding the non-negative function

$$(z - z^{(i)})^t (\alpha I - A A^t) (z - z^{(i)}) \quad (25)$$

to $D(z)$, where α is greater than or equal to the maximum eigenvalue of $A A^t$. So a majorizer of $D(z)$ is given by

$$D(z) + (z - z^{(i)})^t (\alpha I - A A^t) (z - z^{(i)}) \quad (26)$$

and, using the MM approach, the update equation for z is given by

$$z^{(i+1)} = \arg \min_{|z| \leq 1} D(z) + (z - z^{(i)})^t (\alpha I - A A^t) (z - z^{(i)}) \quad (27)$$

$$= \arg \min_{|z| \leq 1} \alpha z^t z - 2 \left(A \left(\frac{2}{\lambda} y - A^t z^{(i)} \right) + \alpha z^{(i)} \right)^t z + K \quad (28)$$

$$= \arg \min_{|z| \leq 1} z^t z - 2 \left(\frac{1}{\alpha} A \left(\frac{2}{\lambda} y - A^t z^{(i)} \right) + z^{(i)} \right)^t z \quad (29)$$

$$= \arg \min_{|z| \leq 1} z^t z - 2b^t z \quad (30)$$

Where

$$b^{(i)} = z^{(i)} + \frac{1}{\alpha} A \left(\frac{2}{\lambda} y - A^t z^{(i)} \right) \quad (31)$$

We need to find $z \in R^M$ minimizing $z^t z - 2b^t z$ subject to the constraint $|z| \leq 1$. Consider first the scalar case:

$$\arg \min_{|z| \leq 1} z^2 - 2bz \quad (32)$$

The solution is given by clipping function:

$$z = \text{clip}(b, 1) := \begin{cases} b & |b| \leq 1 \\ \text{sign}(b) & |b| \geq 1 \end{cases} \quad (33)$$

Note that the vector case (30) is separable --- the elements of z are uncoupled so the constrained minimization can be performed element-wise. Therefore, an update equation for z is given by:

$$z^{(i+1)} = \text{clip} \left(z^{(i)} + \frac{1}{\alpha} A \left(\frac{2}{\lambda} y - A^t z^{(i)} \right), 1 \right) \quad (34)$$

where i is the iteration index.

In summary, the derived iterative algorithm for TV denoising consists of an initialization of Z , say $z^{(0)} = 0$, the update equation (34). Once the update has converged to one's satisfaction, the denoised signal x is given by (19). We call this the *iterative clipping algorithm*

The iterative clipping algorithm for TV denoising can also be written as

$$x^{(i+1)} = y - \frac{\lambda}{2} A^t z^{(i)} \quad (35)$$

$$z^{(i+1)} = \text{clip} \left(z^{(i)} + \frac{2}{\alpha \lambda} A x^{(i+1)}, 1 \right) \quad (36)$$

where $\alpha \geq \max \text{eig}(A A^t)$.

3. Directional Total Variational Algorithm

The block diagram of the proposed algorithm is shown in Fig.1. The proposed algorithm consists of four stages: (1) top-down zig-zag based TV filtering, (2) bottom-up zig-zag based TV filtering, (3) horizontal directional based TV filtering and (4) vertical directional based TV filtering.

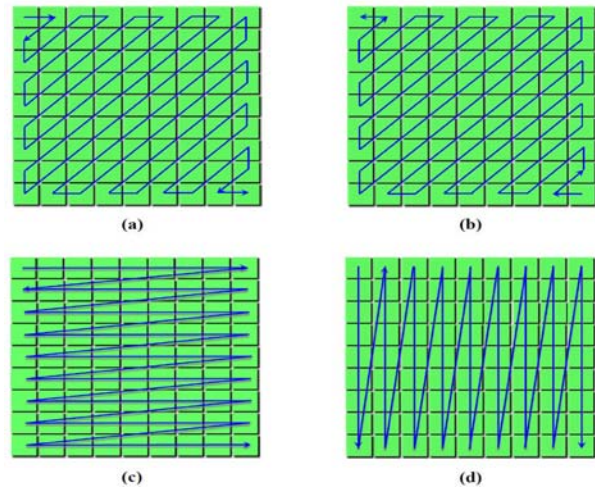


Fig. 2. Block diagram of the directional total variational algorithm. (a) top-down zig-zag based TV filtering, (b) bottom-up zig-zag based TV filtering, (c) horizontal directional based TV filtering and (d) vertical directional based TV filtering

In this section, we describe each step of the proposed algorithm. The TV denoising applied on one-dimensional signals can also be applied for two-dimensional signals by converting the image to 1D array and processing using the TV algorithm. Then the 1D array is converted back to the original 2D form. The steps that are followed to retain the maximum smoothness are by navigating the 2D array through top-down zig-zag, bottom-up zig-zag, horizontal, and vertical directions shown in Fig.2

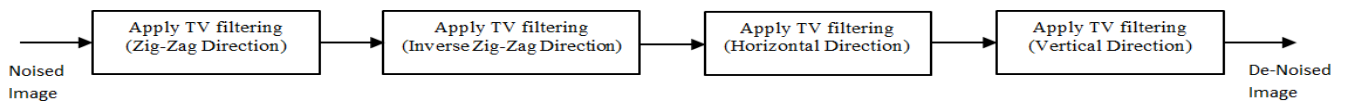


Fig. 1. The Block Diagram of the Proposed System

3.1 Input Noised Image

Noised image is given as input to the algorithm.

Fig. 3 shows the noised Lena image of size 512x512 with SNR ratio 15 dB



Fig. 3. The input noised image.

3.2 Perform top-down zig-zag TV Denoisig

zig-zag pixel reading uses the normal matrix zig-zag navigation procedure for forming the 1D array to be passed for 1D total variation denosing algorithm and the image is denoised which is shown in Fig.4



Fig. 4. Output of TV applied in top-down zig-zag direction.

3.3 Perform bottom-up zig-zag TV Denoisig

The bottom-up zig-zag is the reverse process of top-down zig-zag and is shown in Fig.5



Fig. 5. Output of TV applied in bottom-up zig-zag direction

3.4 Perform horizontal TV Denoising

The horizontal way of pixel reading converts the image into single dimensional array by using the MATLAB matrix and array functionality. The first row of the noised image matrix gets appended by the second row and then by third row and this continuous till the last row. Accomplishment of denoising is done by passing converted 1D array of the signal to the 1D Total variation algorithm and finally reconstructing the image back using the reverse logic used for converting image to 1D array. Fig.6 shows output of Horizontal TV Denoising.



Fig. 6. Output of TV applied in horizontal direction.

3.5 Perform vertical TV Denoising

The vertical pixel reading method is similar as above with input data being the transpose of the original. Fig. 7 shows output of vertical TV Denoising



Fig.7. Output of TV applied in vertical direction.

4. Image Quality Assessment

The performance of the algorithm is tested with different gray scale images. These images corrupted by white Gaussian noise with various signals to noise (SNR) ratios and performance is measured using the following parameter[8]-[18].

4.1 Signal-to-noise-ratio (SNR)

SNR gives the noise suppression quality. Higher the value of SNR, better the noise suppression. If i , ir are the original and denoised images respectively of size m by n then the SNR is given by:

$$\frac{S}{N} = 20 \log_{10} \left(\frac{V_s}{V_n} \right)$$

4.2 Peak-signal-to-noise-ratio (PSNR)

One of the standard performance indexes which is mostly used for 8 bit gray level image.

$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE} \right)$$

4.3 Mean Square Error (MSE)

Mean square error (MSE) is the simplest of the image quality measurement. The larger the value of MSE, poorer the quality of the image. MSE is defined as follow:

$$MSE = \frac{1}{MN} \sum_{ij} (y_{ij} - x_{ij})^2$$

4.4 Laplacian Mean Square Error (LMSE)

This measure is based on the importance of edges measurement. The larger the value of laplacian mean square error (LMSE), the image quality is poor. LMSE is defined as follow:

$$LMSE = \frac{\sum_{j=1}^{M-1} \sum_{k=2}^{N-1} [O\{F(j,k)\} - O\{\hat{F}(j,k)\}]^2}{\sum_{j=1}^{M-1} \sum_{k=2}^{N-1} [O\{F(j,k)\}]^2}$$

4.5 Normalized Absolute Error (NAE)

The larger the value of normalized absolute error (NAE), the quality of the image is poor. NAE is defined as follow:

$$NAE = \frac{\sum_{j=1}^M \sum_{k=1}^N |O\{F(j,k)\} - O\{\hat{F}(j,k)\}|}{\sum_{j=1}^M \sum_{k=1}^N |O\{F(j,k)\}|}$$

4.6 Structural similarity index metrics (SSIM)

If i is original image and ir is denoised image with mean as μ_i , μ_{ir} and variance as σ_i^2 and σ_{ir}^2 , respectively, then SSIM with the default value of $K1$ as 0.01, $K2$ as 0.03 and L as dynamic range of pixel and for 8 bit image, its value is 255, is calculated by using the given equation. If value is closer to 1, then denoised image is closer to original image.

$$SSIM = \frac{(2\sigma_i + c_2)(2\mu_i\mu_{ir} + c_1)}{(\sigma_i^2 + \sigma_{ir}^2 + c_2)(\mu_i^2 + \mu_{ir}^2 + c_1)}$$

4.7 Visual Signal to Noise Ratio (VSNR)

The VSNR, in decibels, is accordingly given by

$$VSNR = 10 \log_{10} \left(\frac{C^2(I)}{VD^2} \right)$$

4.8 Singular Value Decomposition (SVD)

Singular Value Decomposition (SVD) is said to be a significant topic in linear algebra by many renowned mathematicians. SVD has many practical and theoretical values; special feature of SVD is that it can be performed on any real (m, n) matrix. Let's say we have a matrix A with m rows and n columns, with rank r and $r \leq n \leq m$. Then A can be factorized into three matrices:

$$A = USV^T$$

5. Results and Discussion

The performance of the proposed algorithm for two test images including boat and barbara with SNR of 15 dB is shown in Fig.8. The original images are shown in Figs.8(a) and (b). The noisy version of the images are shown in Figs. 8(c) and (d). The output results are shown in Figs.8

(e) and (f). For this testing we set the regularization parameter lambda. The de-noised images are evaluated using subjective and objective matrices. The results are summarized in Table 1. The performance of the proposed algorithm for test images including Peppers and House with SNR of 15 dB is show in Fig.9 The original images are shown in Figs.9 (a) and (b). The noisy version of the images are shown in Figs.9(c) and (d). The output results are shown in Figs.9(e) and (f). For this testing we set the regularization parameter lambda. The de-noised images are evaluated using subjective and objective matrices. The results are summarized in Table 1. The performance of the proposed algorithm for test image FMRI with SNR of 10 dB and 5 dB is show in Fig.10. The original images are shown in Figs.10 a) and (b). The noisy version of the images are shown in Figs.10(c) and (d). The output results are shown in Figs.10 (e) and (f). For this testing we set the regularization parameter lambda. The de-noised images are evaluated using subjective and objective matrices. The results are summarized in Table I.

The performance comparison of the proposed method and the 2D TV denoising method [19],[20] is shown in the Figs.11-14 where (a) represents the original image, (b) represents the noisy image, (c) result of 2D total variation denoising and (d) the result of the proposed method with SNR 15dB and 10dB respectively. The image quality measurement comparing the proposed method and 2D TV denoising method is shown in Table II. The performance comparison of various denoising methods [21]-[24], are shown in Fig.15.

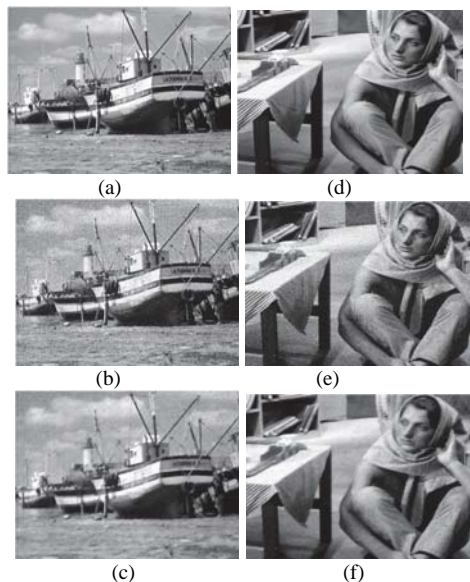


Fig. 8. The performance of the proposed algorithm for different test images with SNR of 15 dB: (a) and (d) are input images; (b) and (e) are

noisy images with SNR of 15 dB; (c) and (f) are outputs of the proposed algorithm

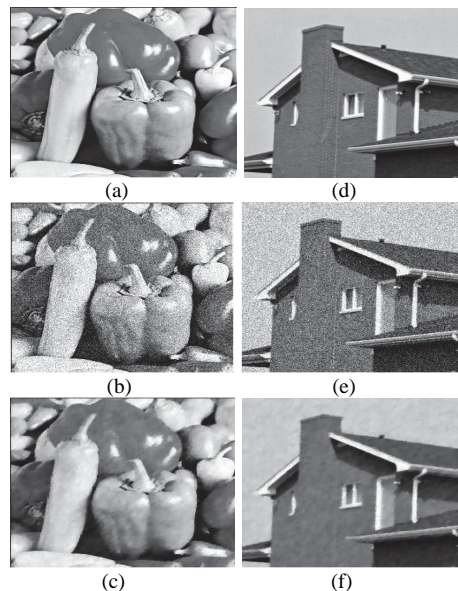


Fig. 9. The performance of the proposed algorithm for different test images with SNR of 15 dB: (a) and (d) are input images; (b) and (e) are noisy images with SNR of 15 dB; (c) and (f) are outputs of the proposed algorithm

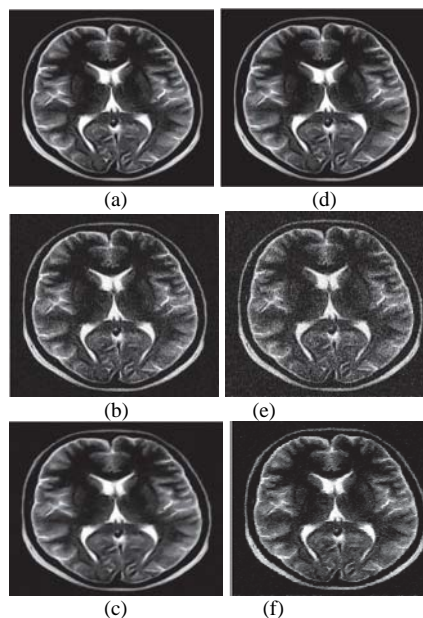








Fig.10. The performance of the proposed algorithm for different test images with SNR of 10 dB and 5 dB: (a) and (d) are input images; (b) is noisy image with SNR of 10 dB and (e) is noisy image with SNR of 5 dB; (c) and (f) are outputs of the proposed algorithm



Fig.11. Performance comparison of One Dimensional and two dimensional TV denoising method with SNR 15 dB: (a) Input image; (b) is the noisy image with SNR 15 dB; (c) result of 2D TV denoising; (d) result of proposed algorithm

TABLE I
 IMAGE QUALITY MEASUREMENT.

Images	SNR	MSE	PSNR	LMSE	NAE	SSIM	VSNR	SVD
	30	127.0133	27.0923	1.7421	0.019	0.9427	15.484	13.6186
	25	127.2957	27.0827	1.7426	0.0193	0.9411	15.4809	13.585
	20	128.0611	27.0566	1.7478	0.0202	0.9358	15.4747	13.4586
	15	134.5335	26.8425	1.7777	0.0234	0.9089	15.4209	11.3769
	10	210.0859	24.9068	2.7139	0.0416	0.7843	14.861	11.5043
	30	167.4629	25.8916	1.4353	0.0284	0.903	14.6176	14.4648
	25	167.3691	25.8941	1.4349	0.0285	0.9025	14.6182	15.6866
	20	168.155	25.8737	1.4392	0.0291	0.8994	14.6145	15.0976
	15	173.8632	25.7287	1.4651	0.0314	0.882	14.569	13.9232
	10	265.6103	23.8884	2.4746	0.0488	0.7839	14.0069	12.1212
	30	116.7587	27.4579	1.3655	0.0229	0.9689	22.266	7.31
	25	116.9914	27.4493	1.3664	0.0233	0.9683	22.2624	7.3133
	20	117.236	27.4402	1.368	0.024	0.9672	22.2606	7.2727
	15	118.2021	27.4046	1.3744	0.0253	0.9634	22.2446	7.0817
	10	125.7833	27.1346	1.41	0.0299	0.9463	22.113	6.0652
	30	13.7658	36.7428	0.0573	0.01	0.965	15.2188	34.9783
	25	14.2139	36.6037	0.0577	0.0104	0.963	15.214	34.2791
	20	15.599	36.1998	0.0594	0.0114	0.9556	15.2078	33.6445
	15	23.4052	34.4377	0.0745	0.0153	0.9201	15.1601	41.5866
	10	109.4648	27.7381	0.7947	0.0351	0.7624	14.5392	39.3764
	30	152.3217	26.3032	0.7867	0.0376	0.8917	15.9831	67.2001
	25	152.095	26.3097	0.7844	0.0377	0.8915	15.9811	66.7994
	20	150.7557	26.3481	0.7729	0.038	0.8901	15.9809	68.6387
	15	151.9315	26.3143	0.7541	0.0397	0.8776	15.9339	60.2889
	10	209.7507	24.9138	1.0305	0.052	0.8052	15.4402	43.5145
	30	169.6276	25.8358	0.5655	0.0464	0.909	12.955	51.8756
	25	169.3645	25.8426	0.5762	0.0464	0.9101	12.9653	50.197
	20	167.8723	25.881	0.6233	0.0462	0.9142	13.0074	52.6736
	15	174.1693	25.7211	0.8676	0.0468	0.9209	13.093	52.9385
	10	326.6404	22.9901	5.2922	0.0633	0.9084	12.8928	46.2461

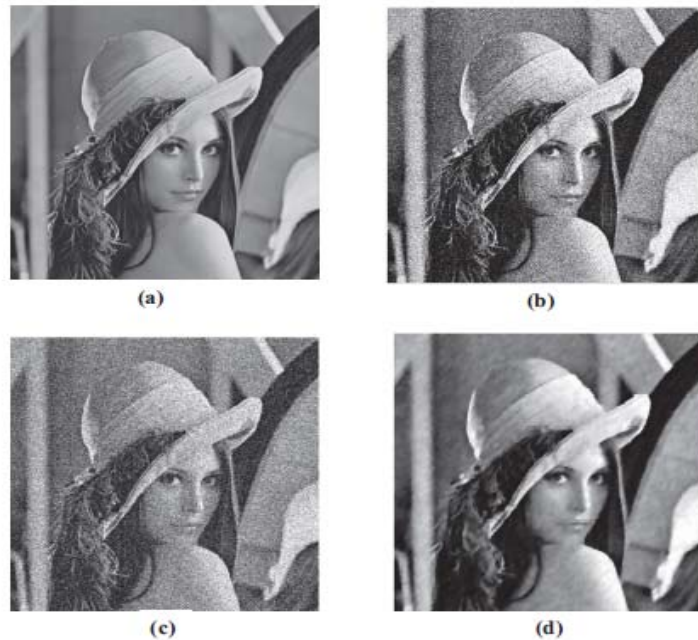







Fig.12. Performance comparison of One Dimensional and two dimensional TV denoising method with SNR 10 dB: (a) Input image; (b) is the noisy image with SNR 10 dB; (c) result of 2D TV denoising; (d) result of proposed algorithm

TABLE II
 1D AND 2D QUALITY MEASURE COMPARISON.

Method	Images	SNR	MSE	PSNR	LMSE	NAE	SSIM	VSNR	SVD	
proposed method		30	127.0133	27.0923	1.7421	0.019	0.9427	15.484	13.6186	
		25	127.2957	27.0827	1.7426	0.0193	0.9411	15.4809	13.585	
		20	128.0611	27.0566	1.7478	0.0202	0.9358	15.4747	13.4586	
		15	134.5335	26.8425	1.7777	0.0234	0.9089	15.4209	11.3769	
		10	210.0859	24.9068	2.7139	0.0416	0.7843	14.861	11.5043	
		30	116.7587	27.4579	1.3655	0.0229	0.9689	22.266	7.31	
		25	116.9914	27.4493	1.3664	0.0233	0.9683	22.2624	7.3133	
		20	117.236	27.4402	1.368	0.024	0.9672	22.2606	7.2727	
	2d tv		15	118.2021	27.4046	1.3744	0.0253	0.9634	22.2446	7.0817
			10	125.7833	27.1346	1.41	0.0299	0.9463	22.113	6.0652
			30	60.3824	30.3217	1.1852	0.012	0.9722	15.7114	7.1378
			25	61.9728	30.2088	1.1997	0.013	0.9697	15.7073	7.0881
		20	72.7845	29.5104	1.2866	0.0185	0.9359	15.6572	6.5512	
		15	147.2278	26.4509	2.3466	0.0379	0.8141	15.2061	9.0558	
		10	558.5544	20.6601	13.1702	0.0856	0.6455	13.6974	14.8095	
		30	93.76	28.4106	1.0456	0.0106	0.9849	18.8067	6.1855	
	25	93.9272	28.4029	1.0469	0.0109	0.9852	18.8067	6.1002		
	20	95.4028	28.3352	1.0597	0.0123	0.9837	18.8073	5.0066		
	15	104.8208	27.9263	1.1269	0.0178	0.9672	18.7879	5.0606		
	10	173.407	25.7401	1.9389	0.0374	0.9051	18.5603	8.5847		

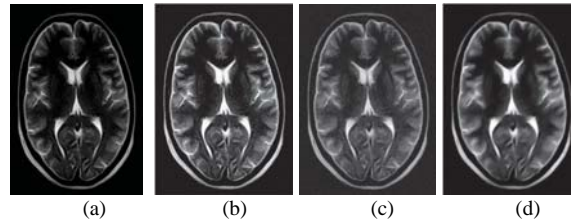


Fig.13. Performance comparison of One Dimensional and two dimensional TV denoising method with SNR 15 dB: (a) Input image; (b) is the noisy image with SNR 15 dB; (c) result of 2D TV denoising; (d) result of proposed algorithm.

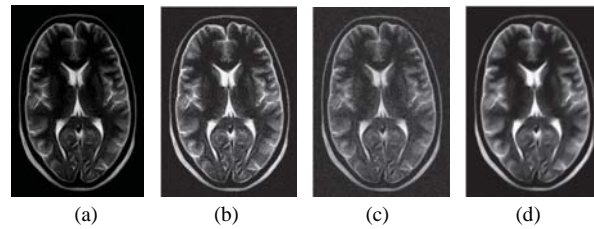


Fig.14. Performance comparison of One Dimensional and two dimensional TV denoising method with SNR 10 dB: (a) Input image; (b) is the noisy image with SNR 10 dB; (c) result of 2D TV denoising; (d) result of proposed algorithm

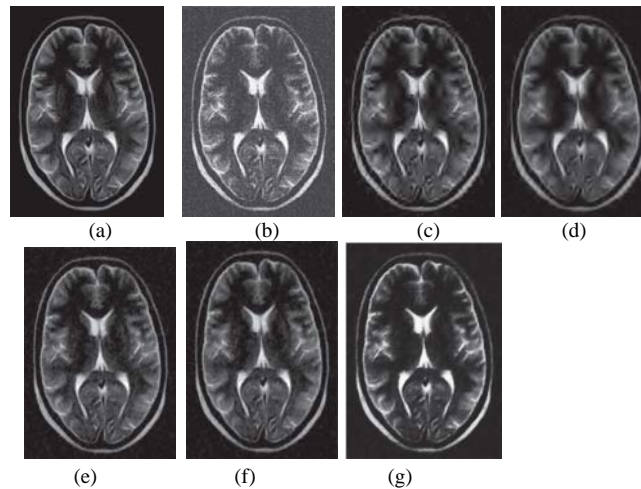


Fig. 15. Performance comparison of various denoising methods.(a) original image; (b) Image with artificial additive Gaussian white noise, with PSNR =2.55 (dB); (C) Denoised image by wavelet hard thresholding; (d) Denoised image by wavelet soft thresholding;(e) Denoised image by TV wavelet with fixed fitting parameter; (f) Denoised image by TV wavelet with variable fitting parameter; (g) Denoised image using proposed method

6 Conclusions

The majorization-minimization procedure along with min-max property is being used. This method can be used for larger images by segmenting the same which can be implemented in Graphical Processing Unit (GPU) processing of images. If computation is not a constraint this technique can be applied for overlapping as well as non-overlapping images. The use of zig-zag helps achieve entire pixel visits on image.

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